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WEIGHT-STRENGTH STUDIES OF STRUCTURES REPRESENTATIVE OF FUSELAGE CONSTRUCTION

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SUMMARY

Weight-strength plots for circular cylindrical shells in bending are presented and discussed. The ring-stiffened shell, the longitudinally stiffened shell, and shells constructed of waffle-like or sandwich-type plates are compared in order to evaluate the effect of type of stiffening on bending strength, and shells of various materials are compared in order to evaluate the effect of material properties.

The sandwich-type stiffening is found to be the most effective stiffening, from a consideration of weight and strength. The ratio of compressive-yield stress to density of material is found to be the most important material parameter for sandwich-type shells. The density of the material is also important and is more important in waffle-like or longitudinally stiffened shells than in sandwich-type shells.

The effect of shear loads on the bending strength of cylinders is considered. The bending strength of cylinders is found to be significantly lowered by the presence of shear loads.

INTRODUCTION

Considerable progress has been made recently in the development of sandwich-type and waffle-like plates. Reliable bonding techniques, the lack of which has retarded the acceptance of sandwich-type plates as aircraft structural components, have been developed for stainless-steel sandwiches (refs. 1 and 2) and are being developed for sandwiches of other materials (refs. 3 and 4). Improvements in chemical milling (ref. 5) and rolling (ref. 6) of waffle-like plates have made the production of such plates feasible.

These developments make desirable a structural evaluation of the various types of fuselage construction. The use of shells with waffle-like or sandwich-type stiffening instead of the more conventional pure-shell (unstiffened shell) or sheet-stringer types of construction for fuselages may extend the range where buckle-free structures can be employed advantageously as well as improve the strength-weight ratio of such structures.

The analytical methods available for making such an evaluation are known to have shortcomings. A simple monocoque in bending, for example, may fail at a fraction of its predicted load. In this study, therefore, existing strength analyses have been modified by "correction factors" which were derived mainly from tests on pure-shell cylinders but which have a small amount of empirical justification for other types of construction.

The structural efficiency of circular cylindrical shells in bending is presented. The walls of the shell may be an unstiffened plate, a longitudinally-stiffened plate, a waffle-like plate or a sandwich-type plate. For shells whose walls are unstiffened plates or sandwich-type plates, the structural efficiency in shear is also given, and the effect of combined loads on these structures is discussed.

SYMBOLS

A	cross-sectional area of column, sq in.
b_H	width of top of hat for hat-section stiffeners, in.
b_S	rib spacing on waffle-like plates, in.
b_W	rib depth on waffle-like plates or stiffener depth on skin-stringer plates, in.
D	plate flexural stiffness per unit width, in-kips
D_1	plate flexural stiffness in longitudinal direction (see ref. 7), in-kips
D_2	plate flexural stiffness in circumferential direction (see ref. 7), in-kips
d	density of material, lb/cu in.
E	Young's modulus, ksi
E_{sec}	secant modulus, ksi
E_1	extensional stiffness in longitudinal direction (see ref. 7), kips/in.

E_2 extensional stiffness in circumferential direction (see ref. 7), kips/in.

H overall depth of sandwich-type or waffle-like plate, in.

h thickness of sandwich-type plate measured between centroids of face sheets, in.

I moment of inertia or shell, $in.^4$

k_s shear buckling coefficient

l ring spacing, in.

M bending moment (see fig. 1), in-kips

N_x compressive load per inch in longitudinal direction, kips/in.

N_{x_o} allowable compressive load per inch in longitudinal direction when only that load is acting, kips/in.

N_y compressive load per inch in circumferential direction, kips/in.

N_{xy} shear flow, kips/in.

N_{xy_o} allowable shear flow when only shear is acting, kips/in.

p internal pressure, ksi

R radius of shell, in.

r_w radius of fillet at base of rib in waffle-like plate, in.

T temperature, ^oF

t_s thickness of skin in pure shell, waffle-like plate or skin-stringer plate; thickness of face sheet in sandwich-type plate, in.

t_w thickness of rib in waffle-like plate or thickness of stiffener in skin-stringer plate, in.

$\bar{t}$ cross-sectional area of shell per inch of circumference expressed as an equivalent thickness, in.

V	shear load on shell (see fig. 1), kips
Z	curvature parameter
$\alpha',\alpha,\beta',\beta$	constants used in studies of waffle-like plates to express the effectiveness of ribs in resisting twisting, transverse bending, shearing, and transverse stretching (see ref. 7)
α_{UL}	upper limit value of α
γ	ratio of failing stress of a cylinder of moderate length in bending to theoretical failing stress of cylinder in compression (see fig. 4)
δ	ratio of core density to face-sheet density of sandwich-type plate
η	plasticity factor
θ	angle of skewed ribbing in waffle-like plates, measured from longitudinal direction, deg
μ	Poisson's ratio
ρ	radius of gyration of column, in.
σ_{cy}	compressive-yield stress of material defined by 0.2-percent-offset method, ksi

EFFICIENCY STUDIES OF FUSELAGES

Preliminary Considerations

The structure considered in this report is a ring-stiffened cylinder of circular cross section. (See fig. 1.) The walls of the cylinder may be an unstiffened plate, a waffle-like plate, a sandwich-type plate, or a plate stiffened by longitudinal stringers. Both bending and shear loadings are considered. Stress-strain curves for the plate materials considered are shown in figure 2, and pertinent material properties are given in table I.

Weight-strength plots are used in the study. Values of $d\,\frac{\bar{t}}{R}$, which measure the weight of the cylinders being considered in pounds per square inch of surface per inch of radius, are plotted against the structural index N_{x_o}/R (for bending loads) or N_{xy_o}/R (for shear loads).

The parameters N_{x_o}/R and N_{xy_o}/R are equivalent for elastic circular cylinders to the parameters M/R^3 and V/R^2 that are sometimes used. For cylinders that fail in the plastic-stress range, the stress distribution around the cylinder must be known or assumed in order that a value may be obtained for the bending moment or the shear load sustained by the cylinders. The parameters M/R^3 and V/R^2 provide a better measure of loading intensity, for this case, if the deviation in stress distribution from the elastic value is taken into account. If the deviation is not taken into account, as is usually the case in efficiency studies, the parameters M/R^3 and V/R^2 are not a true measure of loading intensity and should be replaced by the more descriptive parameters N_{x_o}/R and N_{xy_o}/R. The relation between M/R^3 and N_{x_o}/R for 7075-T6 aluminum-alloy cylinders is given in figure 3.

The weight of reinforcing rings is not included in the weight parameter $d\,\frac{\bar{t}}{R}$. Design data on rings are rather sketchy, and methods of analysis based upon these data are inadequate for use in a weight-strength study where dimensions of structural elements are varied over a large range in order to obtain the lightest structure that will carry a given load. For this reason the weight of the rings was neglected when the studies were made. The relative efficiencies found may be affected somewhat by the proper inclusion of the weight of rings, and this effect is discussed later for the loading conditions and types of loading considered.

Cylinders in Bending

Weight-strength plots are given for cylindrical shells whose walls may be an unstiffened plate, a longitudinally stiffened plate, a waffle-like plate, or a sandwich-type plate. In the computations for these plots, use was made of available theoretical studies on compression cylinders. These studies were modified in order to apply to cylinders in bending by multiplying the computed compressive stresses by a factor γ deduced from tests of pure-shell cylinders in bending (ref. 8) to obtain a value for the extreme fiber stress of a similar cylinder in bending. Values of γ are given in figure 4 as a function of the parameter

$$\frac{R}{\sqrt[4]{\frac{D_1D_2}{E_1E_2}}}$$

which is a generalization of the ratio R/t_s that is usually used for such purposes in the case of the pure-shell cylinders. The general form of the parameter was suggested by the unpublished results of a few tests on waffle-like circular cylinders.

Pure-shell cylinders.- An efficiency plot for pure-shell cylinders of either 7075-T6 aluminum alloy or of AZ31A-H24 magnesium alloy is given in figure 5. The curves are given for cylinders with relatively large ring spacings ($1/2 < l/R < 4$) and were derived with the use of design data of reference 8. The stresses obtained with the use of reference 8 were modified by the plasticity factor developed in reference 9 and presented in design-chart form in reference 10; however, in the present application the stresses were not allowed to be greater than the compressive-yield stress σ_{cy}.

The weight of rings is not included in the parameter $d\,\frac{\bar{t}}{R}$ of figure 5. Cylinder strength is essentially independent of ring spacing except for very small and very large ring spacings which have not been considered herein. Rings are usually used, however, in designs employing this type of construction at spacings not greater than approximately a radius to serve as formers and to improve the shear-strength characteristics of the structure.

The curves labeled $N_{x_o} = \sigma_{cy}\bar{t}$ in figure 5 represent the lightest structures that can be built with the use of the assumptions made. These curves represent structures whose bending stress at the extreme fiber is the compressive-yield stress of the material. The difference in ordinates between the efficiency curves and these curves indicates the potential weight saving that could be achieved if it were possible to design a cylinder to fail at the yield stress for the value of structural index under consideration.

Cylinders with waffle-like walls.- The number of structural variables in a cylinder with waffle-like walls is large. The effects of many of these variables on structural behavior have not been evaluated, and it is beyond the scope of this paper to evaluate all of them. The waffle-like plate considered herein is one termed "isogonigral" in reference 11 and has ribbing which runs at angles of $\pm\theta$ to the axis of the cylinder. This type of plate was studied in reference 12 with regard to its effectiveness in carrying compression and shear loads in flat plates. In that study, ribs running at $\pm 45^{\circ}$ were shown to be quite effective.

In order to estimate what the angle of ribbing should be for waffle-like plates when used on cylinders in bending, a preliminary computation was made on equal-weight cylinders of waffle-like plates on which the angle of ribbing was varied from ±0° to ±90° with the longitudinal axis of the cylinder. The results of this computation are given in figure 6 along with other pertinent dimensions of the cylinders. Here, as with flat waffle-like plates, the ±45° ribbing appears rather effective. The computations were made with the assumption that $\beta = \beta' = 0$; that is, the ribs are assumed effective only insofar as they provide extensional and bending stiffness in the direction of the rib. An idea of the error involved by this assumption can be obtained from a study of figure 7 of reference 7 where the effects of various assumptions on the effectiveness of ribs are compared for pertinent elastic constants. This study indicates that if the effectiveness of the ribs in resisting transverse bending and stretching and in resisting shear and twisting had been taken into account, the ±45° ribbing would compare even more favorably with other angles of ribbing. Figure 6 applies to elastic cylinders. For plastic cylinders changes in the angle of ribbing which increase the effectiveness of the ribs in the axial direction may be desirable for compressive loading. For the practical case where shear loading is acting along with the compressive loading, the ±45° ribbing should again be rather effective.

Another preliminary computation for cylinders with waffle-like walls was made in order to obtain a practical value for the ratio of rib thickness to skin thickness. The results are presented in figure 7. The ordinate of figure 7 can be considered a "shape factor" for the problem under consideration in the same sense that ρ^2/A is the shape factor for columns. For this reason the weight of the cylinders being considered is proportional to the structural index N_{x_o}/R divided by the shape factor; therefore, for a given value of the structural index, improvements in efficiency must be obtained by improving (making larger) the shape factor. The curves are shown dashed in figure 7 at low values of t_W/t_S when b_W/t_W is greater than about 8. This value is probably a limiting value, from a fabrication standpoint; furthermore, larger values of b_W/t_W do not appear desirable for many applications because the ribs of the waffle-like plate will have a lower buckling stress than that of the skin. Again the computations are for $\beta = \beta' = 0$.

On the basis of these preliminary computations, the angle of ribbing was taken as ±45°, and the thickness ratio t_W/t_S was assumed to lie between 1.0 and 2.0 in the final computations. The ratio r_W/b_W was held constant at 1/4 and b_S/t_S and H/t_S were varied in order to find the lightest cylinder with a given value of R/t_S that will carry a given load. The loads that the cylinders sustained before failure were computed with the use of the stability criterion of reference 13 and the assumption (believed to be good for the range of proportions considered) that the transverse shear stiffness was large. The use of this criterion assumes that the coupling actions associated with the use of one-sided stiffening can be neglected. These coupling actions have been neglected in flat-plate studies with good results (see ref. 12) but may be expected to play a more important part in the behavior of curved plates, where buckling is accompanied by a stretching of the neutral surface, than in flat plates, where buckling occurs without a stretching of the neutral surface.

Elastic constants given by the equations of reference 7 and computed with the help of available studies on rib effectiveness were used in the stability criterion. Values of α', β', and β were taken from reference 14 and α was taken as α_{UL} from reference 7. The computed elastic loads were modified by the effectiveness factor γ and by a plasticity factor.

The plasticity factor was applied to waffle-like shells by using the same scheme used in reference 12 for flat waffle-like plates, except the plasticity factor for cylinders in compression from reference 10 was used instead of the value $\frac{E_{sec}}{E}$ used in reference 12.

The results of these computations are given in figure 8 for 7075-T6 aluminum alloy shells and in figure 9 for AZ31A-H24 magnesium-alloy shells. As noted previously, the number of structural variables for a waffle-like plate is large, but not all of them were varied in the calculations for figures 8 and 9. The parameters which were believed to have a substantial influence on structural efficiency were varied, and values for the remaining parameters were assumed or obtained from isolated calculations (see figs. 6 and 7). More comprehensive calculations which involve the variation of all structural parameters would be lengthy and perhaps unjustified in view of the fact that present methods of analysis are approximate and have been supplemented with only a meager amount of experimental data.

Cylinders with sandwich-type walls.- The results of computations of the bending strength of 17-7 PH stainless-steel cylinders with sandwich-type walls are given in figure 10. The particular sandwich considered is one with a honeycomb core constructed from corrugated foils. The density of the core, when the brazing or bonding material is neglected, is 14.5 pounds per cubic foot.

Corrugated-cell cores were considered, even though their shear stiffness for a given density is lower than that of similar noncorrugated-cell cores, because of the resulting ease with which corrugated-cell cores can be formed to the desired contour (see fig. 10 of ref. 1), a feature that is important in fuselage construction. Another feature which appears desirable for the relatively deep sandwiches that may be encountered in fuselage construction and which is inherent in the use of corrugated-cell cores is that some of the problems associated with buckling of the core are eased or eliminated. Core buckling may result from the direct action of applied loads or from crushing loads associated with bending stresses in the sandwich during buckling and may have a detrimental effect on the load-carrying properties of the sandwich.

The choice of a core weighing 14.5 pounds per cubic foot was based upon experience gained in the testing of multiweb beams. An investigation of the strength of multiweb beams in bending (or compression) demonstrated that stresses in the face sheets approaching the yield stress of the face-sheet material could be attained with a ratio of core density to face-sheet density δ (when attachment angles were neglected) as low as 0.013. (See ref. 15.) Extrapolation of this result to sandwiches where the clamping effect is smaller without the attachment angles and where twice as many webs are employed (the equivalent of two sets of webs which run perpendicular to each other) gives a comparable figure for δ of about 0.03. For a steel sandwich the corresponding core density is 14.5 pounds per cubic foot. Trial calculations for sandwiches with lighter cores have been made and indicate a potential means of weight saving. The use of sandwiches employing such cores should be preceded by strength tests, particularly for proportions developing stresses that approach the compressive-yield stress of the material.

The computations for figure 10 were made with the use of reference 13. The elastic loads computed with the use of reference 13 were modified by the effectiveness factor γ of figure 4 and by a plasticity factor. The plasticity factor for a pure shell in compression from reference 10 was used. This factor applies to sandwich cylinders if the stiffness of the sandwich core is large, as was usually the case in the computations made, and applies conservatively to the few cases where small transverse shear corrections were required. For purposes of estimating the transverse shear correction, the ratio of developed length to projected length of the corrugated foil was taken to be 1.2.

The weight of brazing or bonding material is not included in the weight parameter $d\,\frac{\bar{t}}{R}$ of figure 10. This omission has the disadvantage of making comparisons between sandwich construction and other types of construction somewhat indirect but has the advantage of keeping the figure as free from uncertainties as possible. Information on the optimum amount of bonding material required for a sound bond between the face sheets and the core is incomplete at present but appears to be relatively independent of the structural proportions of the plate. For a given value of N_{x_o}/R, therefore, the increase in $d\,\frac{\bar{t}}{R}$ corresponding to the weight of the bonding material will depend upon the size of the fuselage under consideration. There is a loose correlation between the size of the fuselage and the load parameter N_{x_o}/R. Small values of N_{x_o}/R

infer large fuselages, and large values of N_{x_o}/R infer small fuselages. By using the rather average values of $R = 60$ for large fuselages and $R = 15$ for small fuselages, the inclusion of the bonding material was estimated to increase the weight of fuselages about 20 percent at low values of N_{x_o}/R and about 10 percent at high values of N_{x_o}/R.

It is of some interest to investigate the structural configurations associated with various points along a curve of constant R/t_s in figure 10. The only structural parameter that changes along such a curve is H/t_s. If, therefore, R as well as R/t_s is held constant, the change in H/t_s will result from a change in core depth. Increasing the core depth for thin sandwich-type plates effects substantial increases in the moment of inertia of the section and therefore in the load the section will carry before buckling. The increase in weight is, by comparison, small. This increase accounts for the small slope of the curves at low values of N_{x_o}/R. As the sandwich is made deeper and deeper (moving to the right on one of the curves), more and more load is carried by the sandwich before buckling. Eventually, in this process, the stress in the face sheets exceeds the elastic limit of the material, and the slope of the curve increases at a faster rate and finally becomes infinite (with the assumptions made) when the buckling stress equals the compressive-yield stress of the face-sheet material. The values of N_{x_o}/R at which the slope becomes infinite depends on the ratio R/t_s and is given by the equation

$$\frac{N_{x_o}}{R} = \frac{2\sigma_{cy}}{R/t_s} \tag{1}$$

The corresponding value of H/t_s is also dependent upon the ratio of R/t_s. For large values of R/t_s, where stabilization from curvature is small, H/t_s is large, and vice versa.

Sample calculations for sandwiches with load-carrying cores such as corrugated cores (ref. 13) or truss-type cores (a special case of the corrugated core where the width of the flat elements in contact with the face sheets is reduced to a point) have been made for the purpose of comparing the load-carrying ability of these types of construction with that of sandwiches with honeycomb cores. A rather large advantage in favor of sandwiches with truss cores or corrugated cores is the possible elimination of brazing or bonding materials by continuous spot welding of the core and face sheets (ref. 16). The sample calculations indicate that such sandwiches have no advantage in load-carrying ability for lightly loaded fuselages. The fact that the core carries stress does not (in a typical computation) increase the bending stiffness (and therefore the computed elastic load) enough to compensate for the added weight of the heavier core. This disadvantage is partially offset, and for heavily loaded fuselages is overridden, by the fact that the stress-carrying core reduces the detrimental effects of plasticity; that is, a sandwich with a stress-carrying core is less plastic for high loadings than a similar sandwich with a core that does not support a part of the applied load.

Calculations similar to those made for steel cylinders were made for 7075-T6 aluminum-alloy and AZ31A-H24 magnesium-alloy cylinders. Results of the calculations are presented in figures 11 and 12. Although metallurgical bonding processes are not as generally used for these materials as they are for stainless steel, they are used by some manufacturers. Moreover, synthetic adhesives are currently receiving wider acceptance as reliable bonding agents in structural components than previously (see refs. 3 and 4), and in view of these developments the computations for aluminum and magnesium cylinders have been included herein.

Longitudinally stiffened cylinder.- The longitudinally stiffened cylinder chosen for study is one employing hat-section stringers with a width-depth ratio of $b_H/b_W = 0.8$. This particular shaped hat has been shown to be rather effective for use on flat stiffened plates (see ref. 17), and reference 18 indicates that essentially the same stringer properties are involved in the case of curved stiffened plates. Hat-section stringers were chosen because they appear to be quite popular with designers of fuselage structures and because it is believed that their behavior under load is easier to compute than that of Z-section stringers which are also popular. Sometimes Z-section stringers fail in the twisting mode which is not considered to be an important mode for hat-section stringers and is not considered herein.

The structure considered was analyzed with the use of the stability equation for orthotropic cylinders in compression given in reference 13 and an efficiency study for stiffened curved plates similar to the one for flat stiffened plates given in reference 19. Variations which facilitate considerably application of this type of study for stiffened flat plates are given in references 20 and 21. In the application of the stability equation (ref. 13) the assumption was made that the transverse shear stiffness of the stiffened plate was large and that the coupling actions associated with the use of plates with one-sided stiffening could be neglected.

The stability equation of reference 13 often simplifies considerably when a particular structure is considered. In the case of the longitudinally stiffened shell it was found convenient to solve for the load in terms of pertinent structural dimensions. The result is a form similar to that given in reference 18 where the load is given as the sum of two terms; one term gives the load that a geometrically similar flat stiffened panel will carry, and the other term gives a correction to the flat-panel load that accounts for the curvature of the shell. The particular formula used gives buckling loads which are slightly lower than those computed with the use of reference 18 when the ratio of radius to thickness of the shell is small.

The curvature term in the buckling equation was modified by the effectiveness factor γ given in figure 4. In addition, the shell was assumed to be simply supported at the rings, and the plasticity factor for columns from reference 22 was assumed to apply.

In order for the procedures just outlined to apply, the attachment between the stringers and skin must be such that wrinkling or interrivet buckling does not occur. (See ref. 23.) Furthermore, structural proportions must be such that local buckling does not occur prior to failure. This latter condition was conservatively maintained by keeping the width-thickness ratio of the various elements making up the shell cross section so small that their critical stress was greater than the failing stress of the shell, the elements being assumed to be flat and to have simply supported unloaded edges.

The condition that local buckling does not occur prior to failure limits somewhat the efficiencies attainable when the loading is small. (See, for instance, ref. 20.) Computations for shells on which the skin buckles early were not made, however, because the validity of such computations would be questionable. Even the computations made herein for the relatively simple case of a shell that does not buckle locally prior to failure should have supplementary experimental confirmation.

One additional restriction was imposed in the computations made. Preliminary computations indicated that the depth of stringers tended to be rather large for maximum structural efficiency when R/t_S was small and the ring spacing was large. In order to keep reasonably realistic proportions to the structures finally analyzed, the stringer depth was arbitrarily limited to 5 percent of the radius of the shell $\left(b_W/R < 1/20\right)$.

The results of computations for longitudinally stiffened shells are given in figure 13. A rather large change in efficiency is indicated with changes in ring spacing for this type of structure when the weight of the rings is not taken into account. In order to obtain some idea of the relative location of the various curves of l/R if the weight of rings had been considered, a computation using Shanley's criterion for ring design was made. (See ref. 24.) Some Z-section rings of sturdy section $\left(\frac{b_W}{t_W} \approx 20\right)$ were used. The resulting area of the rings was small, and the assumption was made that an area equal to the area of the rings was required in order to attach the rings to the fuselage wall between stringers. When the weight of the cylinders including the weight of the rings was plotted against the structural index, the three curves for various ratios of l/R fell in a relatively narrow band with the curve for $l/R = 1.0$ about 10 percent higher than the lower envelope of the various curves of l/R over most of the structural index. For this reason, the curve for $l/R = 1.0$ of figure 13 is used later in this paper when comparing this type of structure with other types whose strength is not changed appreciably by changes in ring spacing. Such a comparison would seem to be valid because, as mentioned earlier, rings are usually used on the other types of construction at spacings not greater than a radius.

Discussion.- Numerous comparisons can readily be made with the use of the weight-strength plots presented herein. One of the more interesting is that of comparing the effectiveness of various types of stiffening for fuselages in bending. In order to expedite this comparison, weight-strength plots for four different types of construction are given in figure 14 for circular shells of 7075-T6 aluminum alloy. The curve for the pure shell and the envelope curve for the waffle-like shell have been taken directly from figures 5 and 8. The curve given for the sandwich-type shell is from 10 percent to 20 percent higher, depending upon the value of N_{x_o}/R, than the envelope curve of figure 11 to account for the weight of the bonding agent, as discussed previously; and the curve for the longitudinally stiffened shell is the one given for $\frac{l}{R} = 1.0$ in figure 13.

Figure 14 indicates that waffle-like shells and longitudinally stiffened shells are nearly equivalent, from a weight and strength consideration, for the range of loading intensities N_{x_o}/R considered. The pure shell is considerably heavier and the sandwich-type shell is considerably lighter than any of the other types of construction considered. The comparisons given apply to shells that are buckle-free at ultimate load. This restriction has little or no effect on the results shown except for the longitudinally stiffened shell which would appear slightly better at the lower values of N_{x_o}/R considered and could be extended to much lower values of N_{x_o}/R if the restriction were lifted.

The comparison given in figure 14 indicates that sandwich-type construction has a decided weight advantage over the other types of construction considered. All of the weight advantage shown, however, may not be realized in practice because joints between skin panels and between the skin and other members such as rings are much heavier for sandwich-type construction, according to contemporary design procedures, than for the other types of construction considered. On the other hand, sandwich construction is relatively new, and improvement in such detailed design problems seems a likelihood.

The part that material properties play in determining the weight of fuselages is demonstrated in figure 15 for sandwich-type construction and in figure 16 for waffle-like construction. Sandwich-type structures can be stabilized, by proper design, to withstand high stresses in the face sheets for a large range of values of $\bar{N}_{x_o}/R$; consequently, the curves shown in figure 15 represent structures for which the stresses in the face sheets are $0.85\sigma_{cy}$ or greater and are therefore highly dependent upon the ratio σ_{cy}/d. The magnesium and steel considered herein have relatively low ratios of compressive-yield stress to density when compared with the ratio for 7075-T6 aluminum alloy, and sandwiches from these two materials are never as light as sandwiches from 7075-T6 aluminum alloy. If the three materials had the same value of σ_{cy}/d (they have approximately the same value of E/d, which is also a prerequisite) and stress-strain curves of the same shape, structures made from magnesium would be the lightest and structures made from steel would be the heaviest of the three materials considered.

Waffle-like shells are not as easily stabilized to withstand high stresses as are sandwich-type shells. The location of the weight-strength curve for the waffle-like shells is, therefore, less dependent upon the ratio σ_{cy}/d than is the curve for sandwich-type shells. For this reason magnesium structures can be designed which are lighter than aluminum structures for small loading intensities $\left(N_{x_o}/R\right)$ for waffle-like shells (fig. 16) but not for the sandwich-type shells (fig. 15).

Cylinders in Shear

The aircraft fuselage derives its bending stresses, for the most part, from transverse loads. A problem arises, therefore, concerning the importance of the effect of shear stresses on the strength (and subsequently the weight) of fuselages. This effect is often assumed to be small in weight studies, but this assumption may not be justified in some cases. In order to obtain a better appreciation of the effect that shear stresses have on the strength of fuselages, some computations are made with the use of available methods of analysis.

The first step in these computations is the determination of the strength of fuselages subjected to a "pure shear" loading. The computations are made for cylinders with the use of theories applicable to cylinders loaded with a uniform shear flow around the circumference (torque load). The shear-stress distribution of interest varies sinusoidally around the circumference. This distribution is caused by radial shear loads on the cylinder and is accompanied by a normal (bending) stress distribution. For many engineering applications, however, it is convenient to compute the bending and "shear" buckling loads separately and to use an interaction curve to find the buckling load for a cylinder subjected to combined bending and shear. If the loading were physically possible, a cylinder subjected only to a sinusoidal shear stress distribution would presumably sustain a greater maximum shear flow before buckling than it does in torsion. (An analogous problem exists for normal stresses between the cylinder in bending and the cylinder in compression, for instance.) Tests on torsion cylinders have demonstrated that available torsion theories (ref. 25) are about 15 percent unconservative on the average, but the use of such theories here to compute buckling loads due to a sinusoidal stress distribution is expected to give conservative predictions. This expectation is substantiated by the investigation of reference 26.

Pure-shell cylinders.- Weight-strength plots for pure-shell cylinders of 7075-T6 aluminum alloy and of AZ31A-H24 magnesium alloy are given by figure 17. The curves are given for a ratio of ring spacing to radius of 1.0 and apply to cylinders simply supported at the rings. The curves were derived with the use of reference 25 and a plasticity factor for flat plates in shear from reference 27. Here, paralleling the procedure used earlier in the computations for cylinders in bending, the shear stress at buckling was not allowed to be any greater than $\sigma_{cy}/\sqrt{3}$. (See ref. 27.)

The buckle pattern for cylinders in shear is such that changes in ring spacing result in corresponding changes in allowable stress. The change is rather small, however, and cylinders of fuselage proportions with $l/R < 1.0$ are not likely to be more efficient than cylinders with $l/R = 1.0$ if the weight of rings is considered. The fact was shown previously, for example, that the efficiency of longitudinally stiffened shells in bending is rather insensitive to changes in ring spacing when the weight of the rings is considered. The allowable stress in that case was nearly inversely proportional to l^2. For the present case the allowable stress is much less sensitive to changes in ring spacing and is inversely proportional to $\sqrt{l}$. The efficiency of cylinders in shear, therefore, is expected to be less than that suggested by figure 17 when smaller ring spacings are used.

Other design considerations in addition to the weight-strength consideration discussed herein play a part in determining the most desirable ring spacing for a given design; for instance, reference 28 indicates that small ring spacings in pressurized fuselages reduce the likelihood of catastrophic decompression failures.

Cylinders with sandwich-type shells.- A comprehensive analysis of sandwich-type shells in torsion similar to that of reference 13 for compression is not available. Analyses of so called "long" sandwich cylinders have been made, but these analyses are too conservative when applied to shells of fuselage proportions. The analysis of reference 29, for instance, predicts buckling loads for the proportions considered in this investigation that are as little as 10 percent of that predicted by theories which properly account for ring spacing.

The analysis of reference 25 for pure-shell cylinders is applicable to the sandwich-type cylinders considered herein if the transverse shear stiffness of the core is large and if a proper definition of terms is made. The plots of k_S against Z of that reference, for instance, can be applied to isotropic sandwich-type shells if the ordinate and abscissa are taken as

$$k_S = \frac{N_{xy} l^2}{\pi^2 D} \tag{2}$$

and

$$Z = \frac{l^2}{Rh}\sqrt{\frac{1 - \mu^2}{3}} \tag{3}$$

where

$$D = \frac{E t_S h^2}{2\left(1 - \mu^2\right)}$$

and μ is Poisson's ratio for the face-sheet material.

Weight-strength plots for stainless-steel, aluminum, and magnesium sandwich-type cylindrical shells with a ratio of core density to face-sheet density of 0.03 are given by figures 18 to 20. Again the plasticity factor for flat plates in shear from reference 27 was used.

The plots of figures 18 to 20 are practically unaffected by the assumption that the shear stiffness of the core is large. Some sample calculations have been made which indicate that the transverse shear effects are small for cylinders of the proportions considered herein. The calculations were made by using the equilibrium equation of reference 30 and a procedure similar to that used in reference 25 for pure-shell cylinders.

Cylinders in Shear and Bending

A convenient way of expressing the relative importance of shear and bending in the aircraft fuselage is by the ratio N_x/N_{xy} or by the equivalent for elastic cylinders M/RV. This ratio has the physical significance of being the distance measured in radii from the station of interest to the centroid of the external loads. Typical design values of N_x/N_{xy} for aircraft fuselages lie between 1.5 and 6.0 and, as a rough approximation, the value of N_x/N_{xy} of 1.7 may be considered the dividing line between fuselages designed primarily for shear and fuselages designed primarily for bending.

For purposes of estimating the weight of fuselages under combined loads the fuselage may be divided into four equal quadrants, two of which are designed from bending considerations and the other two from shear considerations. This scheme is mentioned in reference 24 and appears quite feasible because over 80 percent of the bending moment is carried by the bending material defined in this manner. A similar statement can, of course, be made in the case of shear. The effect of changes in shear area on the effectiveness of the cylinder in resisting bending is shown graphically in figure 21. The ordinate of the graph of figure 21 is the ratio of the moment of inertia of a cylinder with variable skin thickness to the moment of inertia of a cylinder with a skin thickness t around the complete circumference. It is, therefore, a measure of the effectiveness of the cylinders considered in resisting bending loads.

Reference 31 shows experimental data for cylinders subjected to pure bending and torsion which are rather uniformly distributed on either side of a circular interaction curve. Less interaction is expected for the loading case considered herein than for the case considered in reference 31 because, in the present case, the bending stresses are a maximum in the region where the shear stresses are a minimum, and vice versa. The use of the circular interaction curve for the loading case considered herein is therefore expected to be somewhat conservative. A study of the data of reference 26 substantiates this expectation. When these data are plotted with the use of the methods outlined herein to calculate the pure-shear and pure-bending loads, nearly all the data fall on the outside of a circular interaction curve. A considerable amount of scatter exists in the test results of reference 26 for various reasons. (See ref. 8.) The data are therefore not very suitable for establishing an interaction curve and, when more reliable data are available, they may indicate that a more optimistic interaction curve than a circle can be used.

Figure 22 shows that the portion of the circular interaction curve that is of practical interest in fuselage design is small. This figure was constructed from the results of a study of cylinders of the type previously discussed, where two quadrants are designed from bending considerations and two quadrants are designed from shear considerations.

The bending and shear portions of the cylinder were of the same type of construction, either unstiffened or sandwich-type plates, and of the same material. The range will become larger if different materials and different types of construction are employed in a given design. The results shown in figure 22 are given in a different type of plot in figure 23.

The results of some computations for cylinders of 7075-T6 aluminum-alloy sandwich-type plates subjected to combined shear and bending are given in figure 24. The computations were made with the use of figures 3, 11, 19, 21, and 23, where the corrections implied by figures 3 and 21 were applied in an approximate manner; that is, the corrections were assumed to be small enough that an iterative procedure was unwarranted. The results given in figure 24 apply to cylinders of constant thickness. The presence of the shear load increases cylinder weight very little for $\frac{N_x}{N_{xy}} = 6.0$ but increases the weight 35 to 40 percent for $\frac{N_x}{N_{xy}} = 1.5$. Figure 25 indicates the weight saving that can be made by dropping the restriction of uniform thickness around the circumference. A 15 to 20 percent saving is indicated for a value of $\frac{N_x}{N_{xy}} = 6.0$, but a negligible saving is indicated for $\frac{N_x}{N_{xy}} = 1.5$. The negligible saving is because this design is near the borderline mentioned earlier of cylinders on one side designed primarily from bending considerations and cylinders on the other side designed primarily from shear considerations.

Other Considerations

The weight-strength charts in this report were derived for rather idealized structures and loading conditions. For instance, actual structures may derive a stabilizing effect from internal pressure, if the fuselage is pressurized, or from the fuselage floor, if design conditions are such that the floor of the fuselage is in compression; furthermore, the structures may be subjected to elevated temperatures or irregular stress distributions resulting from large cutouts or concentrated loads that may have a detrimental effect on the load-carrying ability of the structure. Effects of the fuselage floor and irregular stress distribution are largely a matter of individual design for each specific aircraft; however, the effects of internal pressure or of a uniform elevated temperature can be considered in a general study.

The principal effect of internal pressure is to load the shell in tension according to the following equations:

$$N_x = -\frac{p}{2} R \tag{4}$$

$$N_y = -pR \tag{5}$$

Equation (4) shows that a pressurized shell can be loaded with a larger bending moment before compressive failure than a similar nonpressurized shell. This effect can be accounted for in design when weight-strength charts for nonpressurized shells are employed by adjusting the structural index by the amount $\Delta \frac{N_{x_o}}{R} = -\frac{p}{2}$. For conventional transport aircraft this adjustment is small (about 0.004 ksi) but can become important for a missile or an aircraft whose fuselage is primarily an integral, pressurized fuel tank.

The circumferential load component N_y may provide a stabilizing effect and raise the buckling load of the shell. (See, for instance, ref. 32.) In this respect, the effect of increasing internal pressure is similar to that of reducing irregularities or eccentricities in the shell. The stabilizing effect is, therefore, encountered only in designs in which $\gamma < 1.0$ (see fig. 4) and usually does not amount to much if the design is determined on a weight-strength basis; that is, the proportions of designs which determine the location of the envelope curve of a weight-strength plot are usually such that $\gamma = 1.0$.

Figure 26 shows the envelope curves for cylinders of 17-7 PH stainless-steel sandwich-type plates at room temperature (80° F) and at 600° F. The curve for 600° F was obtained as described previously for the room-temperature computations, except a stress-strain curve typical of the material at the elevated temperature was employed (see fig. 2). Although the elevated temperature has the effect of decreasing the yield stress by about 17 percent and Young's modulus by about 8 percent, the value of N_{x_o}/R at any given value of $d \frac{\bar{t}}{R}$ has been decreased by only about 15 percent. This result exemplifies the characteristic, mentioned earlier, that optimum designs in sandwich-type structures are heavily dependent upon the ratio σ_{cy}/d. The reason that the change in N_{x_o}/R is less than the corresponding change in yield stress is because the proportions of the two structures being compared are somewhat different. In each case optimum proportions have been found for the values of E/d and σ_{cy}/d under consideration, and less stabilizing material is required to stabilize the sandwiches of the lower-strength material.

The structural efficiency of shells of other materials or of materials at elevated temperatures can be extrapolated from the results of computations made herein; for instance, sandwich-type shells of RC-130-A titanium (see fig. 1 and table I) are expected to be about as efficient as shells of 7075-T6 aluminum alloy at room temperature and nearly as efficient as shells of 17-7 PH stainless steel at 600° F. A new experimental titanium alloy (Ti-6Al-4V) is reported to possess considerably better ratios of compressive-yield stress to density both at room temperature and at elevated temperatures than the titanium alloy considered herein (see ref. 33) and consequently may be suitable for use in all types of construction both at room temperature and at elevated temperatures.

CONCLUDING REMARKS

Weight-strength plots, suitable for use in strength and weight analyses in the preliminary design stage of aircraft construction, have been presented for cylindrical shells in bending. Of the types of stiffening considered, sandwich-type stiffening was found to be the most effective, and no decided advantage was found between longitudinal and waffle-like stiffening for minimum-weight designs within the range of structural configurations considered. The ratio of compressive-yield stress to density of material was found to be the most important material parameter for sandwich-type shells while the density of the material was found to be equally as important for shells with longitudinal or waffle-like stiffening.

The effect of shear loads on the bending strength of cylinders was considered, and the bending strength was found to be significantly lowered by the presence of shear loads.

Langley Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 11, 1957.

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TABLE I

PROPERTIES OF MATERIALS

Material	d, lb/cu in.	μ	T = 80° F			T = 600° F		
			E, ksi	σ_{cy}, ksi	σ_{cy}/d, kip-in/lb	E, ksi	σ_{cy}, ksi	σ_{cy}/d, kip-in/lb
17-7 PH stainless steel	0.278	0.28	30,000	180	647	27,500	150	540
RC-130-A titanium	.170	.33	17,000	130	765	13,000	85	500
7075-T6 aluminum alloy	.101	.32	10,500	72	713	------	---	---
AZ31A-H24 magnesium alloy	.064	.35	6,500	25	391	------	---	---

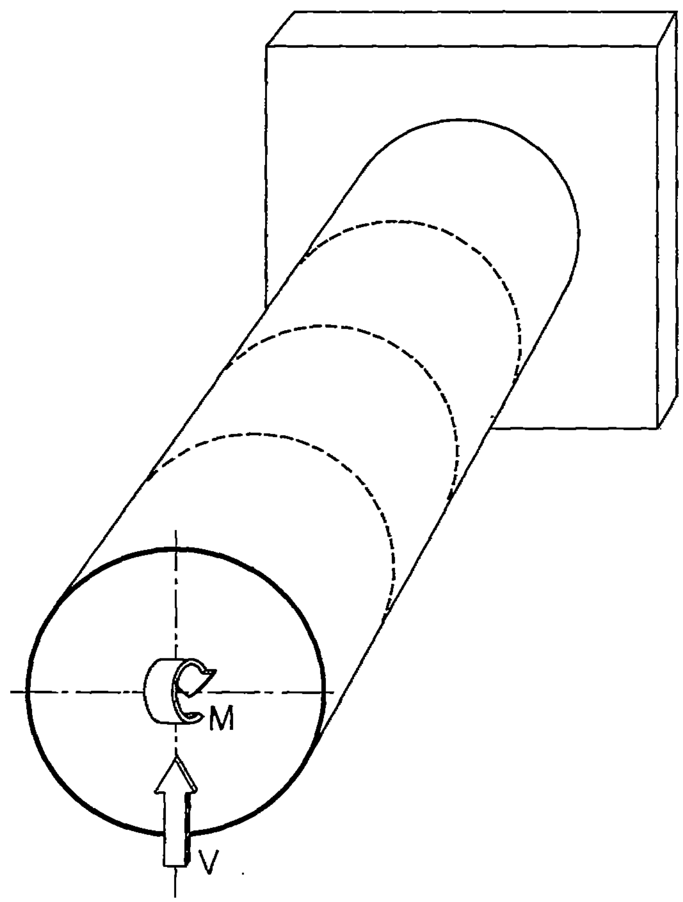

Figure 1.- Cylinder subjected to bending and shear loads.

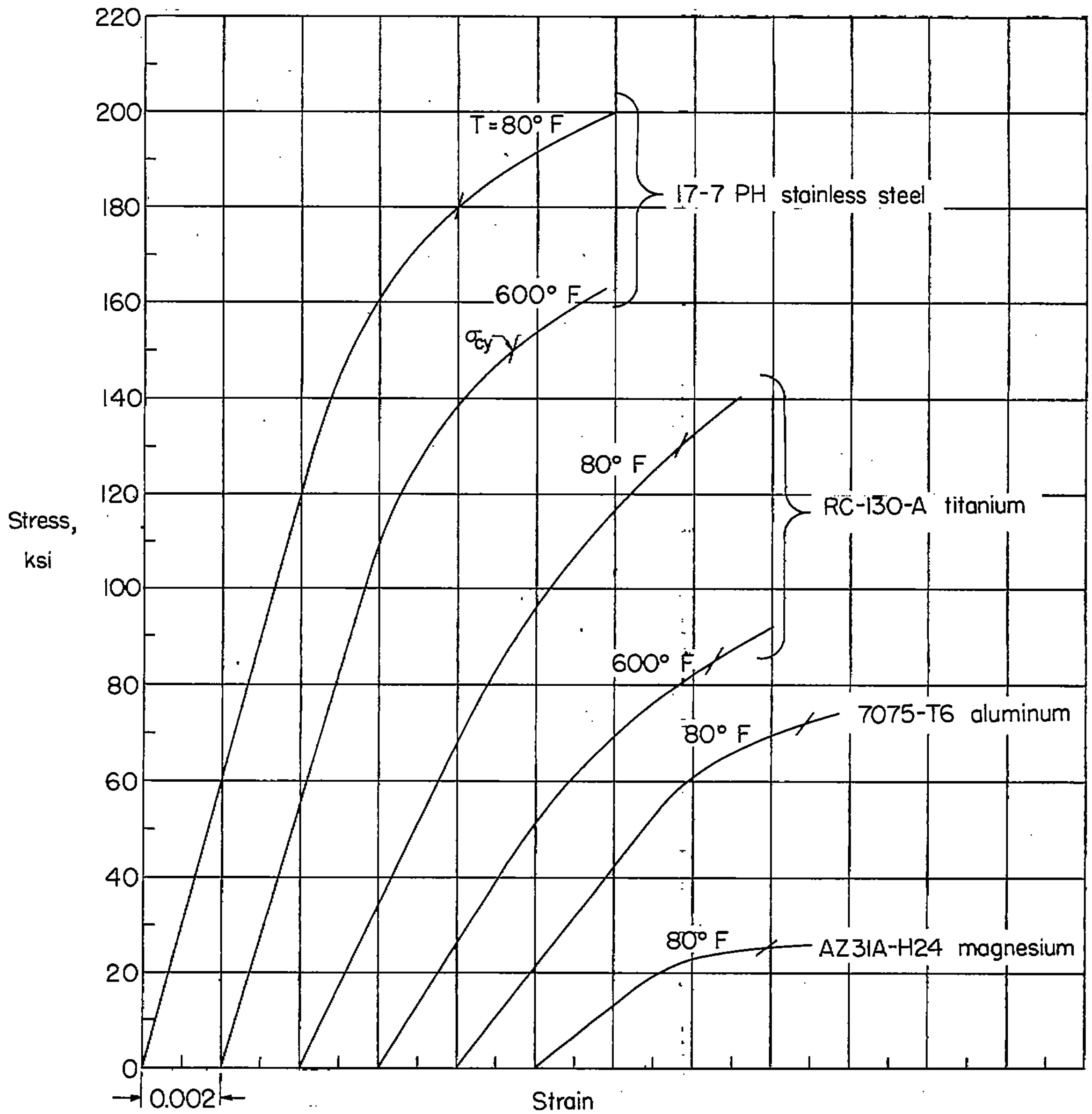

Figure 2.- Stress-strain curves used for analysis.

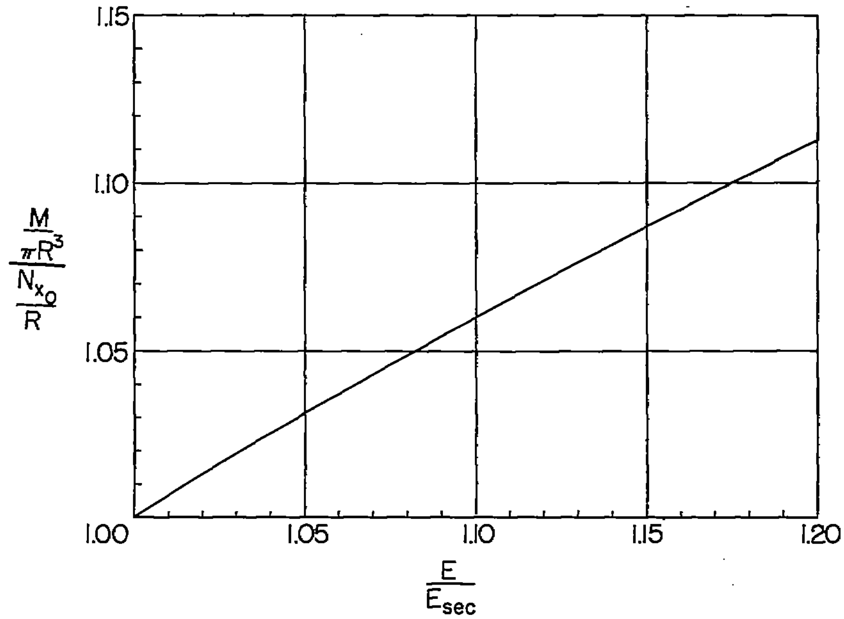

Figure 3.- Relation between two structural indices for plastic cylinders in bending.

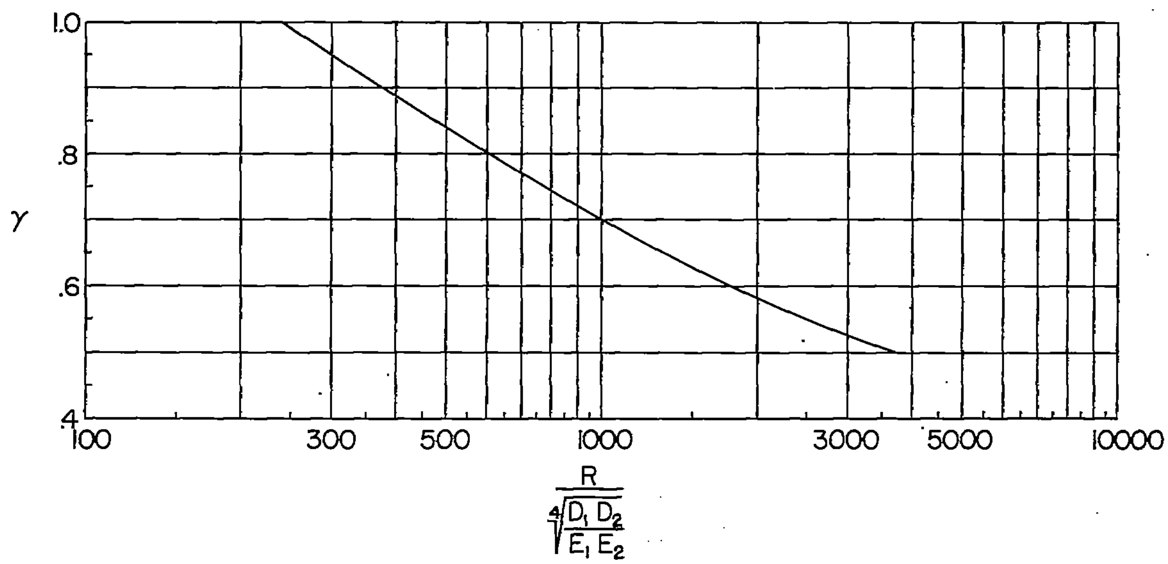

Figure 4.- Ratio of failing stress of a moderately long cylinder in bending to theoretical failing stress of the cylinder in compression.

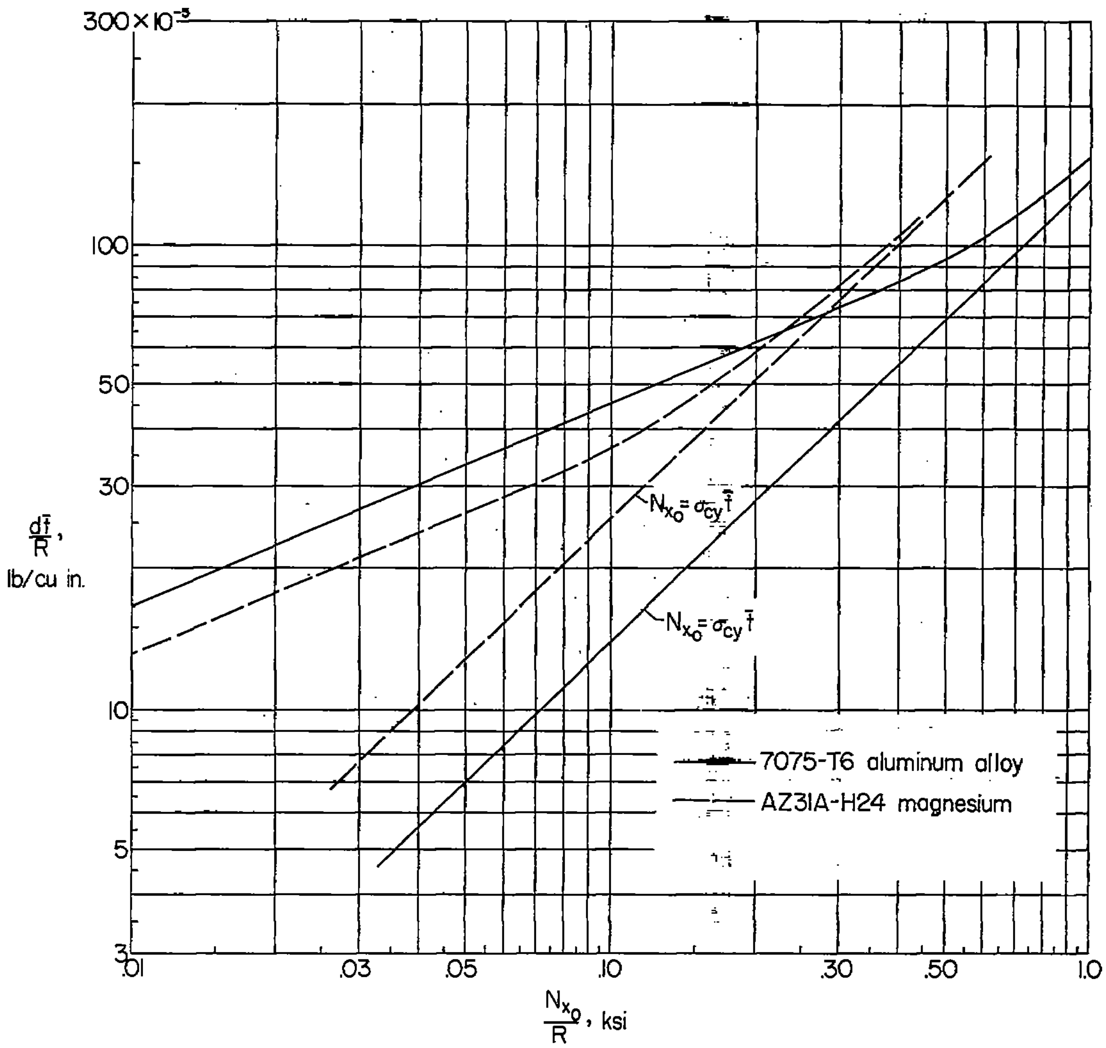

Figure 5.- Weight-strength plot of 7075-T6 aluminum-alloy and AZ31A-H24 magnesium-alloy cylinders subjected to bending stresses.

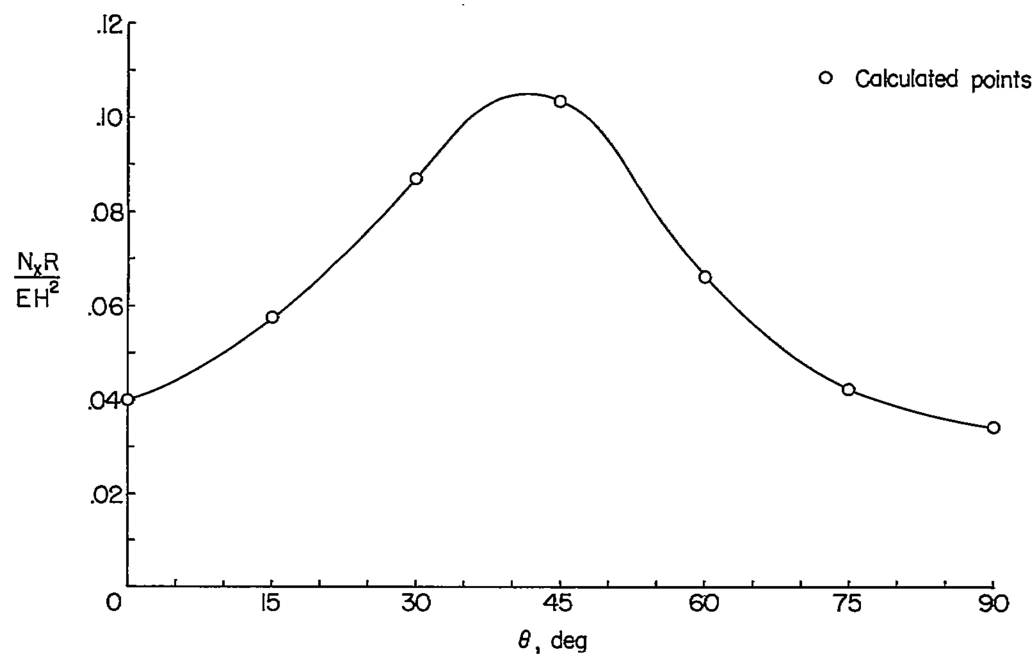

Figure 6.- Effectiveness of "isogonigral" waffle-like stiffening as a function of skew angle. $R/t_S = 1{,}000$; $b_S/t_S = 20$; $H/t_S = 5$; $t_W/t_S = 2$; $r_W/b_W = \frac{1}{2}$.

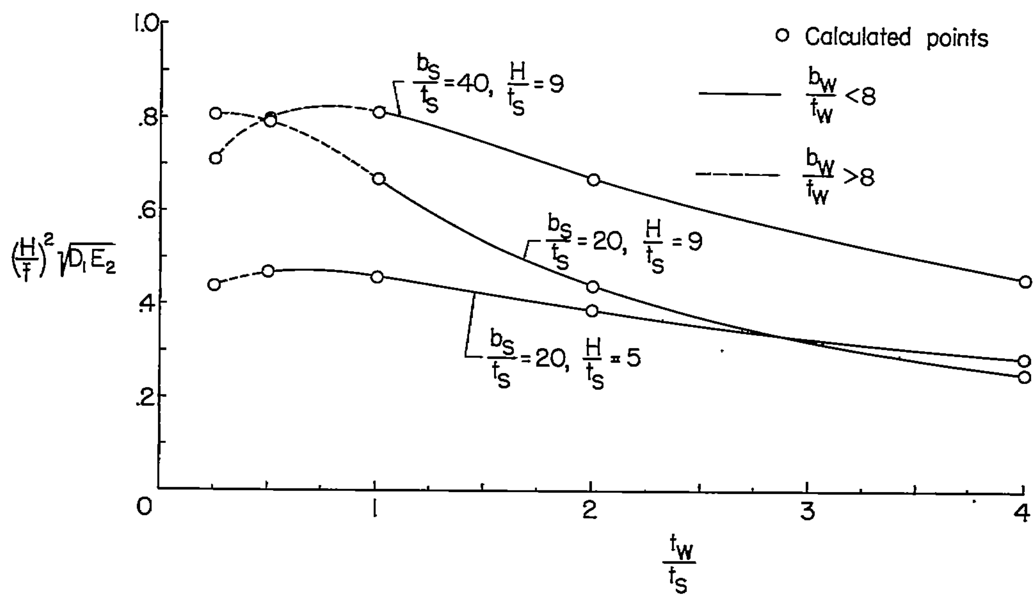

Figure 7.- Effect of changes in the parameter t_W/t_S on the strength of some waffle-like cylinders. $\theta = 45^{\circ}$; $r_W = 0$.

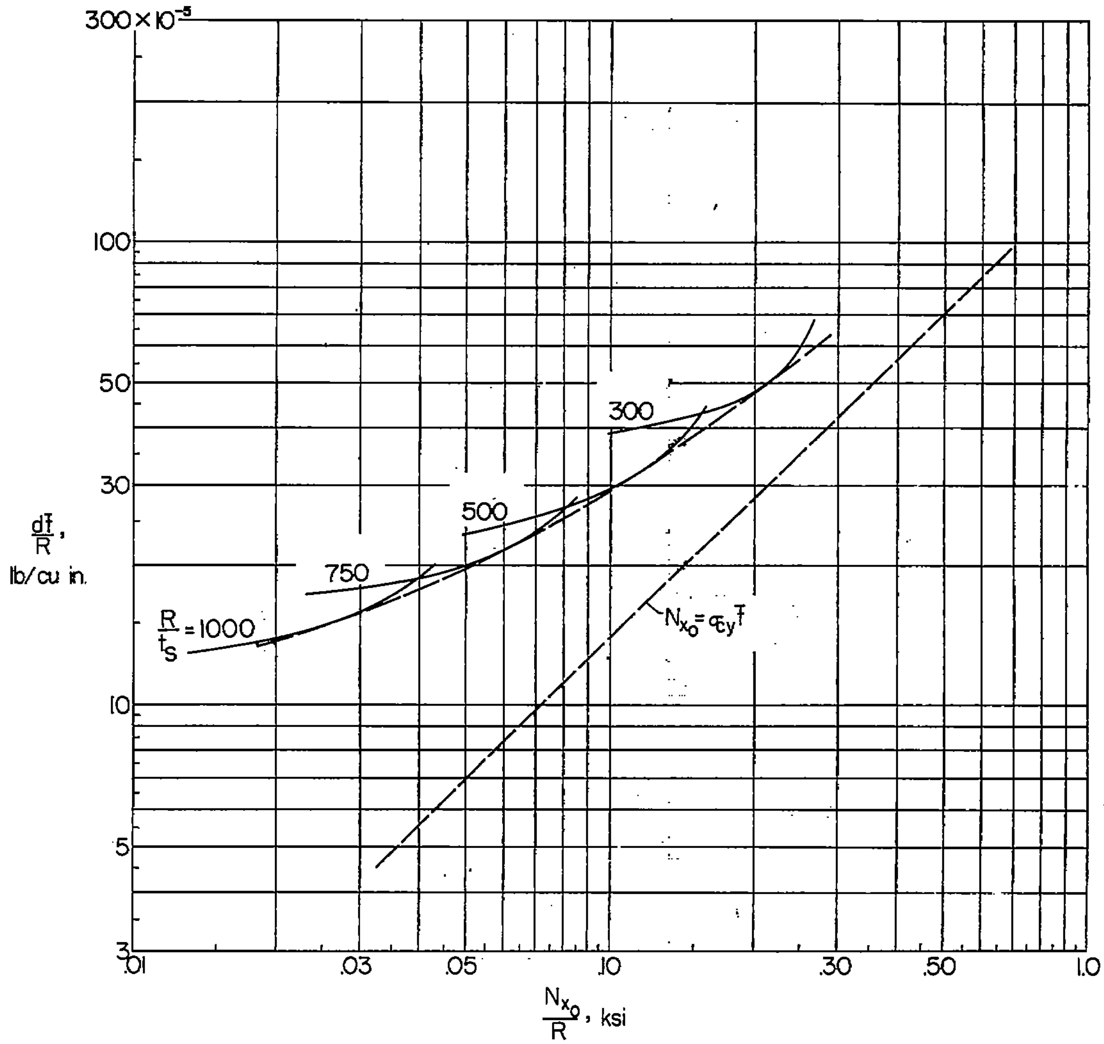

Figure 8.- Weight-strength plot of 7075-T6 aluminum-alloy waffle-like cylindrical shells subjected to bending stresses.

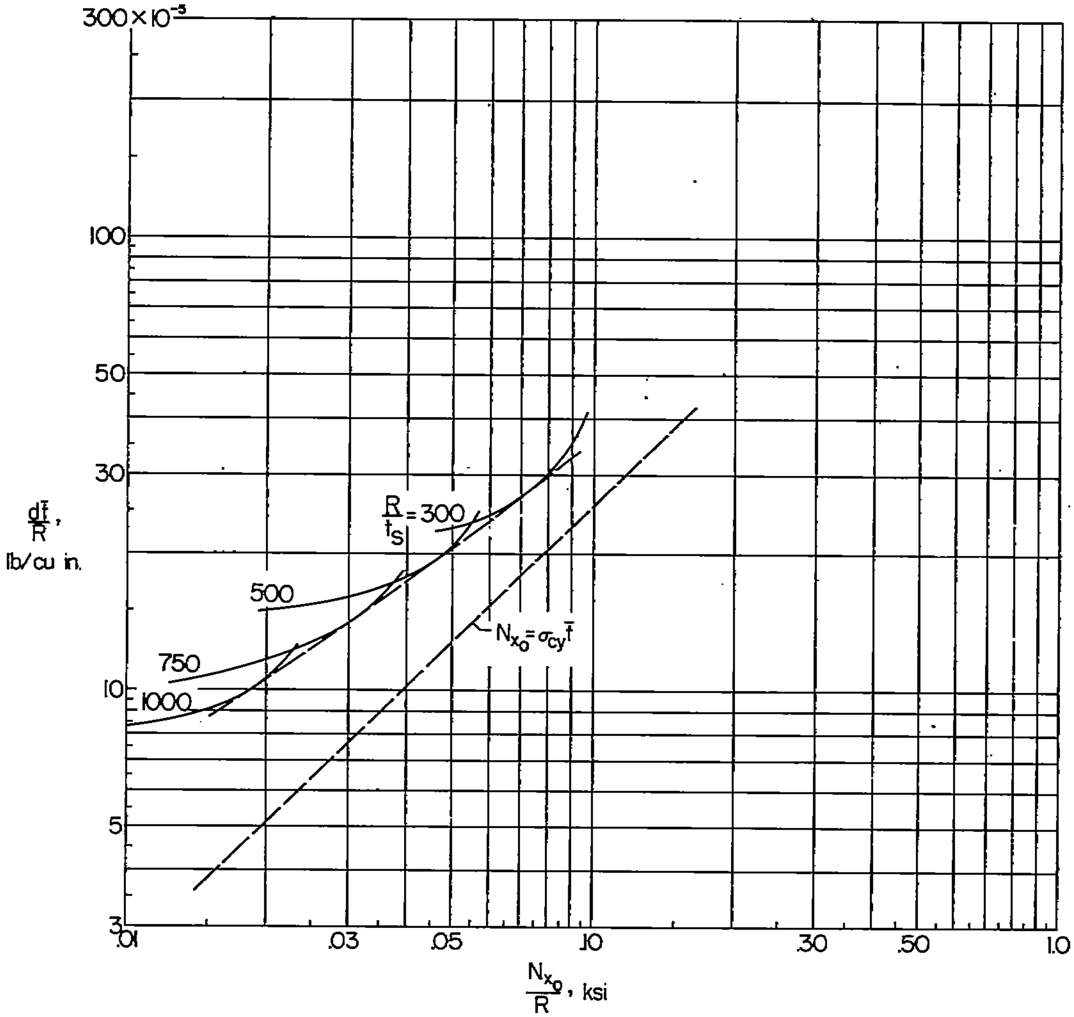

Figure 9.- Weight-strength plot of AZ31A-H24 magnesium-alloy waffle-like cylindrical shells subjected to bending stresses.

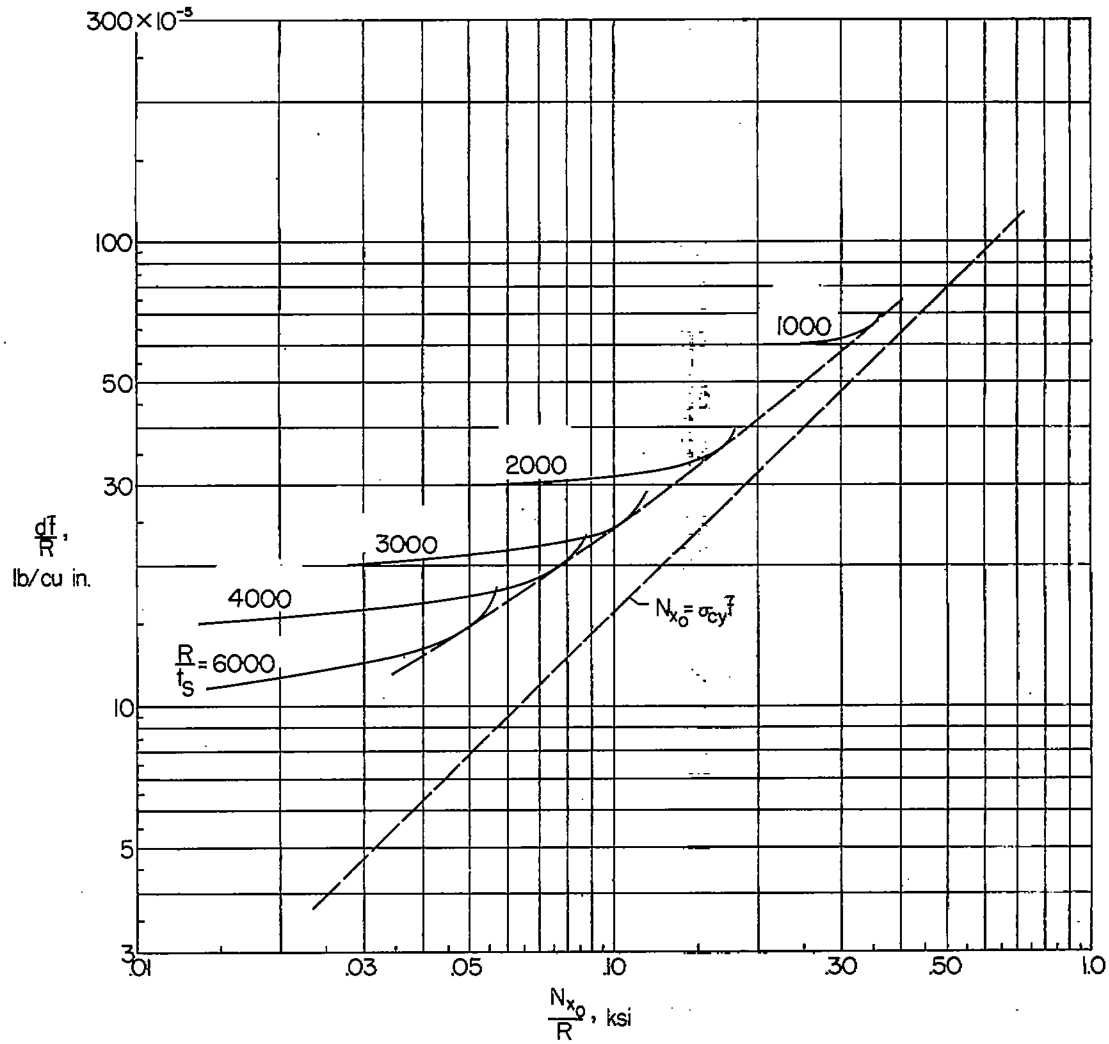

Figure 10.- Weight-strength plot of 17-7 PH stainless-steel sandwich-type cylindrical shells subjected to bending stresses. $\delta = 0.03$.

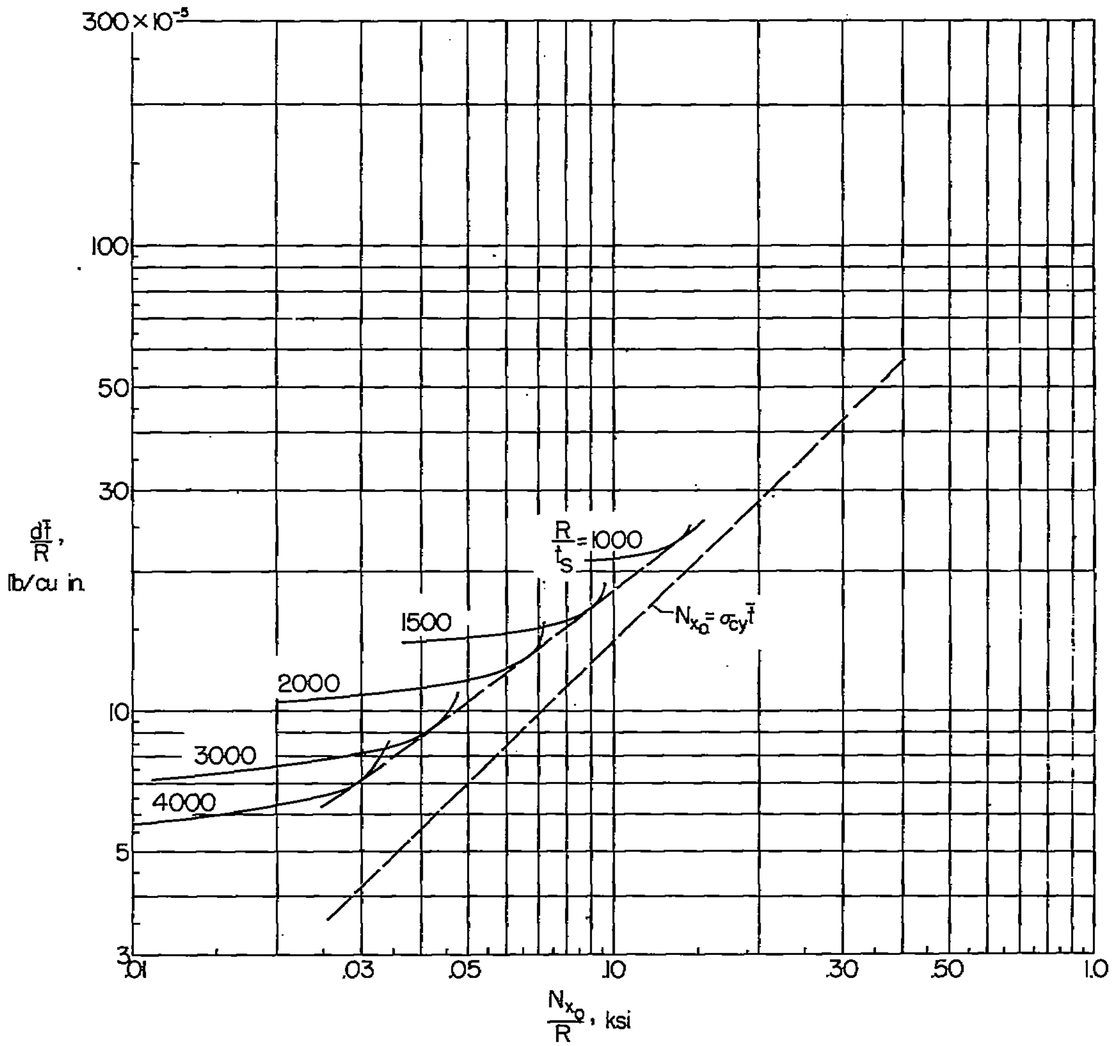

Figure 11.- Weight-strength plot of 7075-T6 aluminum-alloy sandwich-type cylindrical shells subjected to bending stresses. $\delta = 0.03$.

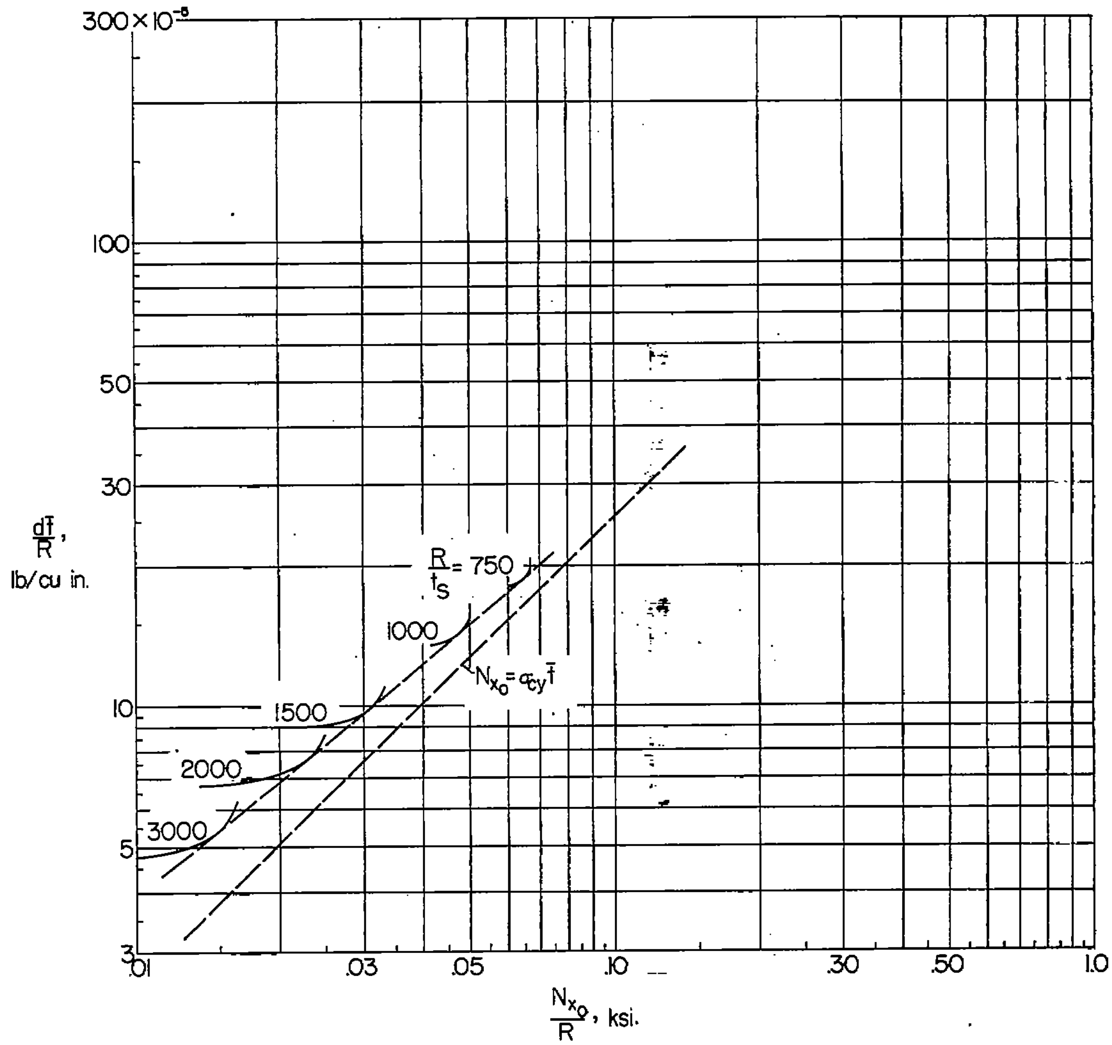

Figure 12.- Weight-strength plot of AZ31A-H24 magnesium-alloy sandwich-type cylindrical shells subjected to bending stresses. $\delta = 0.03$.

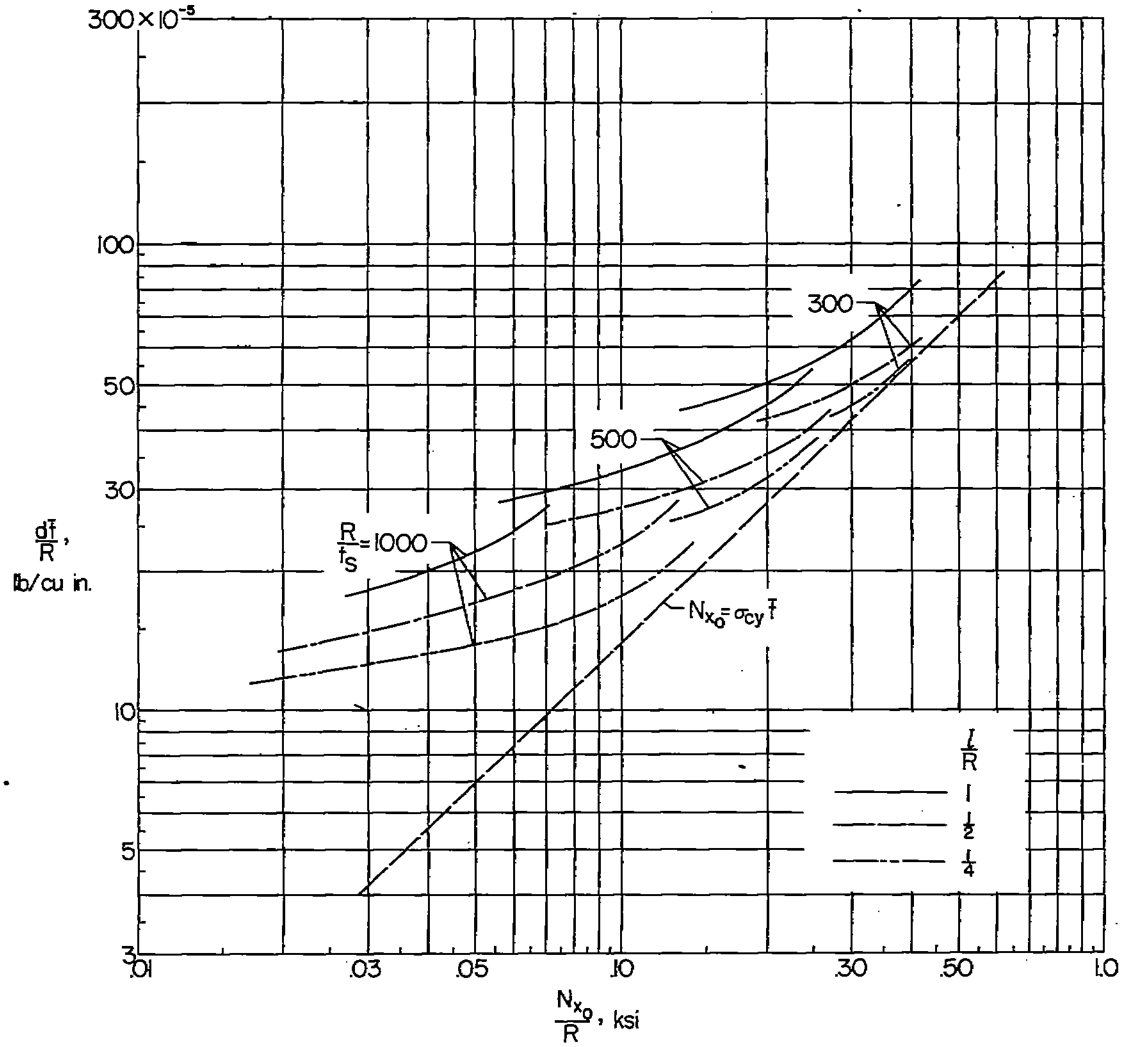

Figure 13.- Weight-strength plot of 7075-T6 aluminum-alloy longitudinally stiffened cylindrical shells subjected to bending stresses.

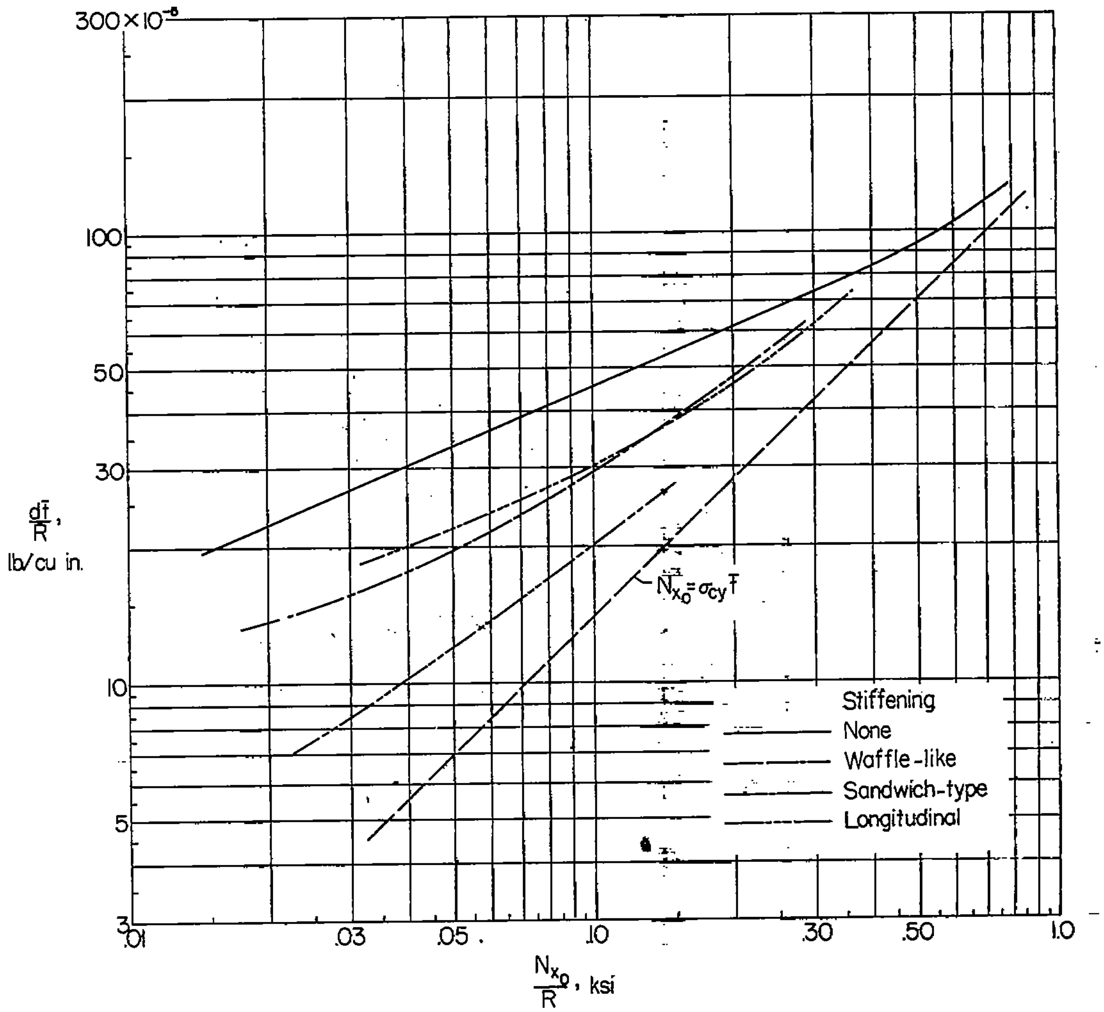

Figure 14.- Weight-strength comparisons of 7075-T6 aluminum-alloy cylindrical shells with various types of stiffening. Shells are subjected to bending loads.

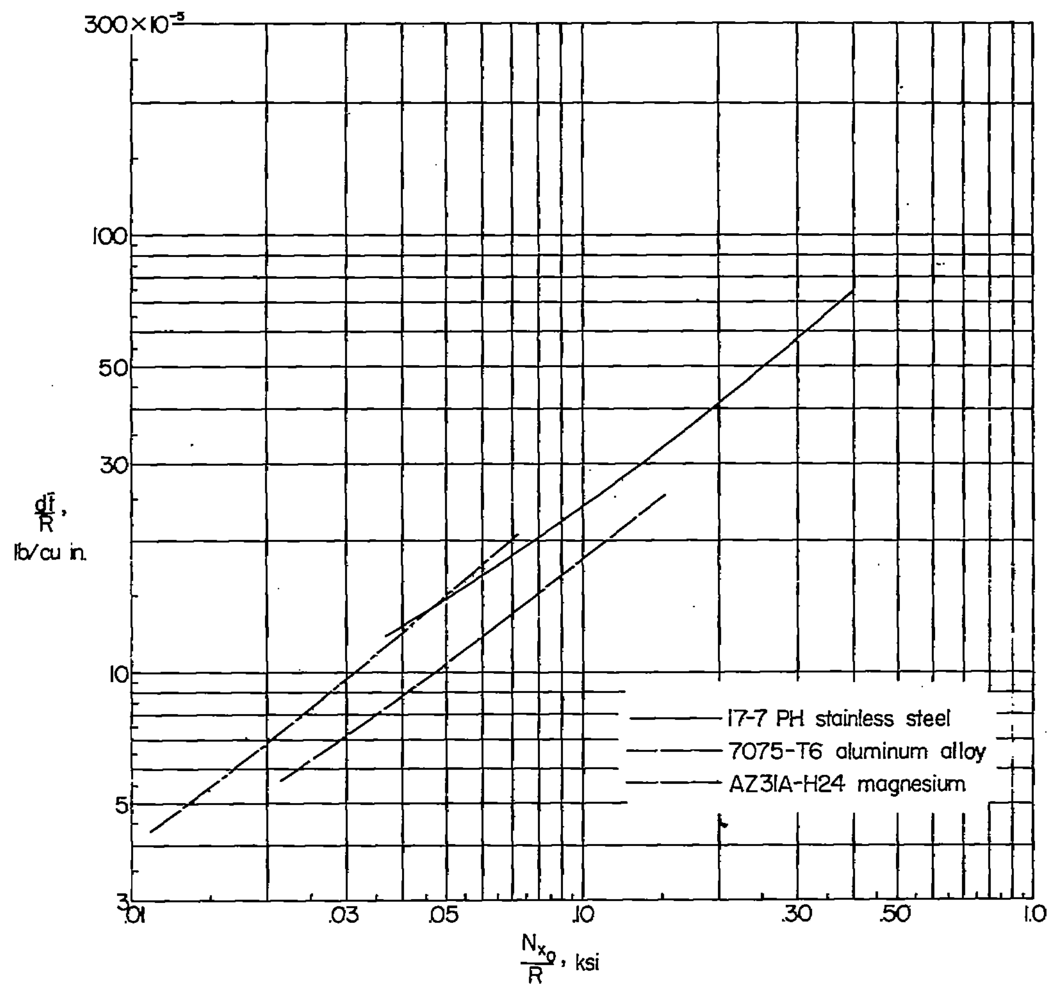

Figure 15.- Weight-strength comparisons of cylindrical shells with sandwich-type walls of various materials. Shells are subjected to bending loads.

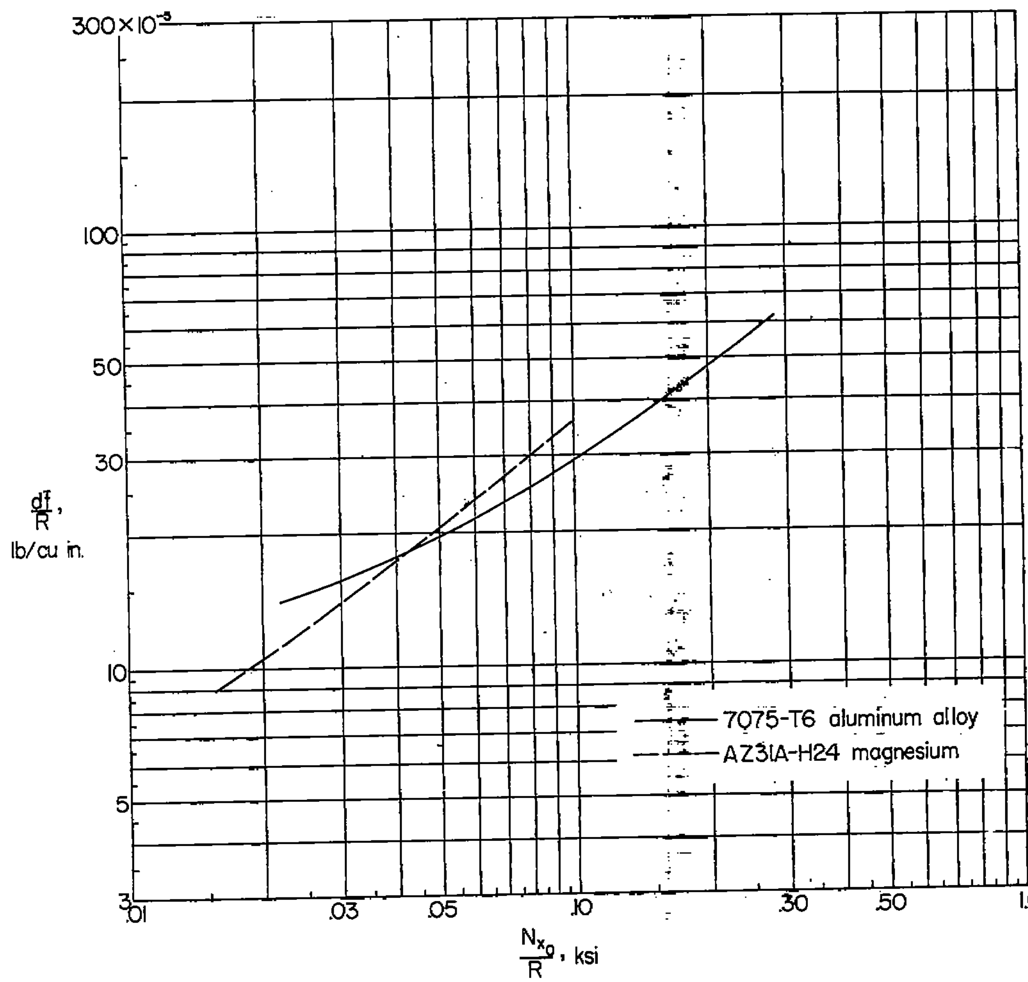

Figure 16.- Weight-strength comparisons of cylindrical shells with waffle-like walls of 7075-T6 aluminum alloy and AZ31A-H24 magnesium alloy. Shells are subjected to bending loads.

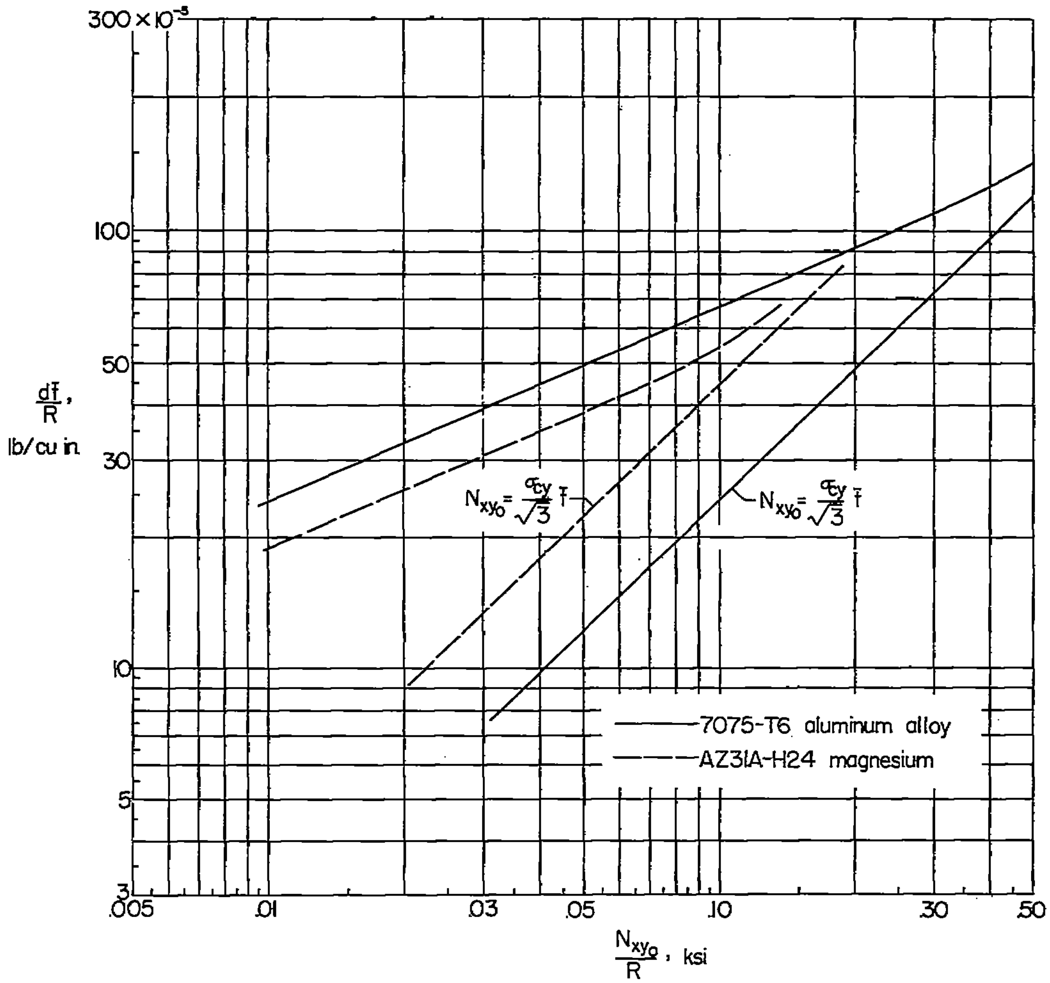

Figure 17.- Weight-strength plot of 7075-T6 aluminum-alloy and AZ31A-H24 magnesium-alloy cylindrical shells subjected to shear stresses. $l/R = 1.0$.

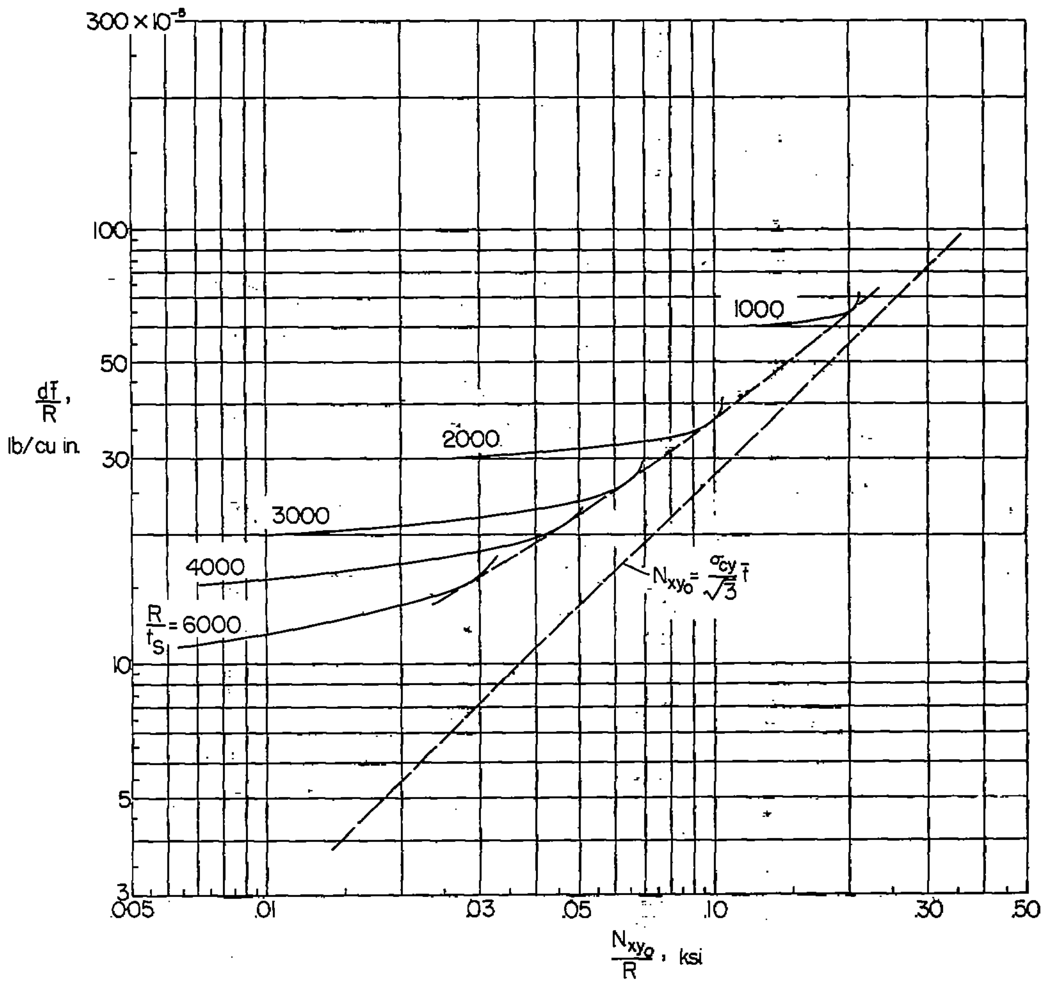

Figure 18.- Weight-strength plot of 17-7 PH stainless-steel sandwich-type cylindrical shells subjected to shear stresses. $\delta = 0.03$; $l/R = 1.0$.

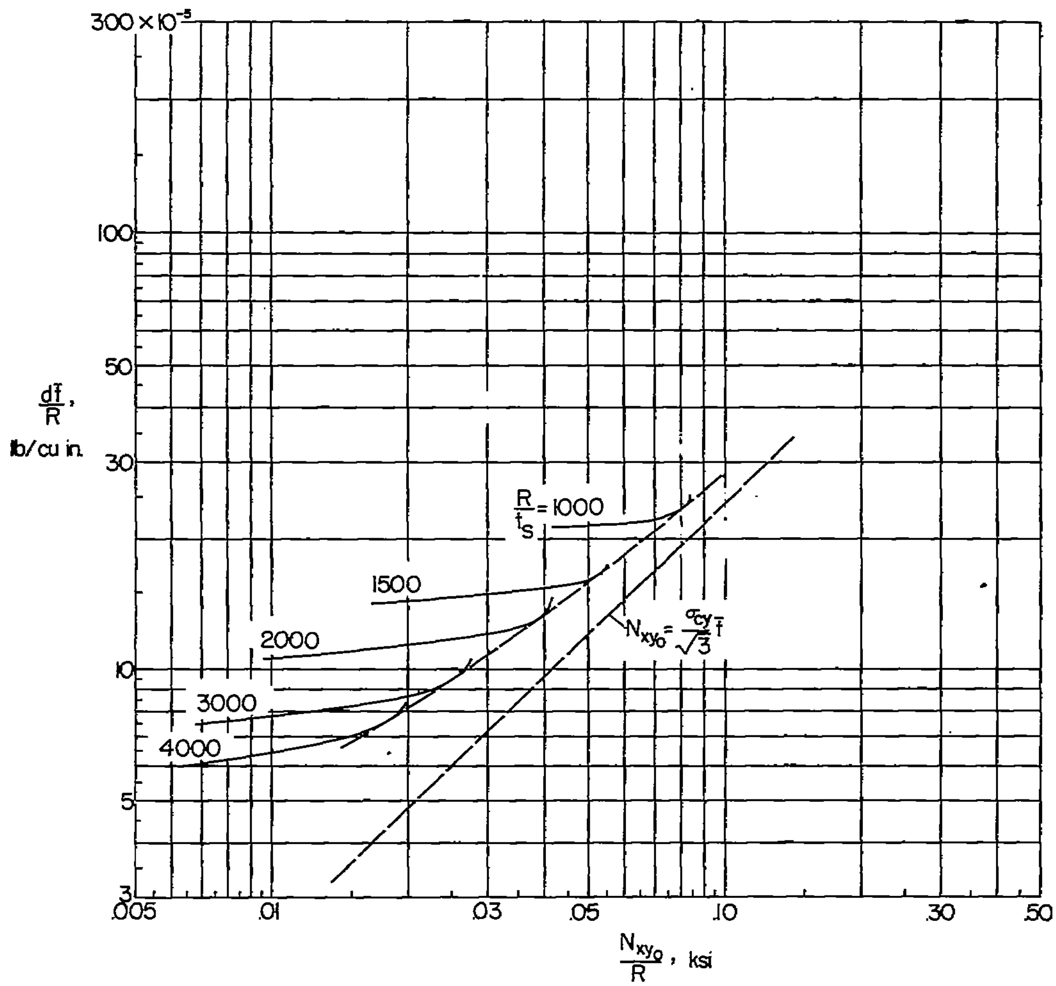

Figure 19.- Weight-strength plot of 7075-T6 aluminum-alloy sandwich-type cylindrical shells subjected to shear stresses. $\delta = 0.03$; $l/R = 1.0$.

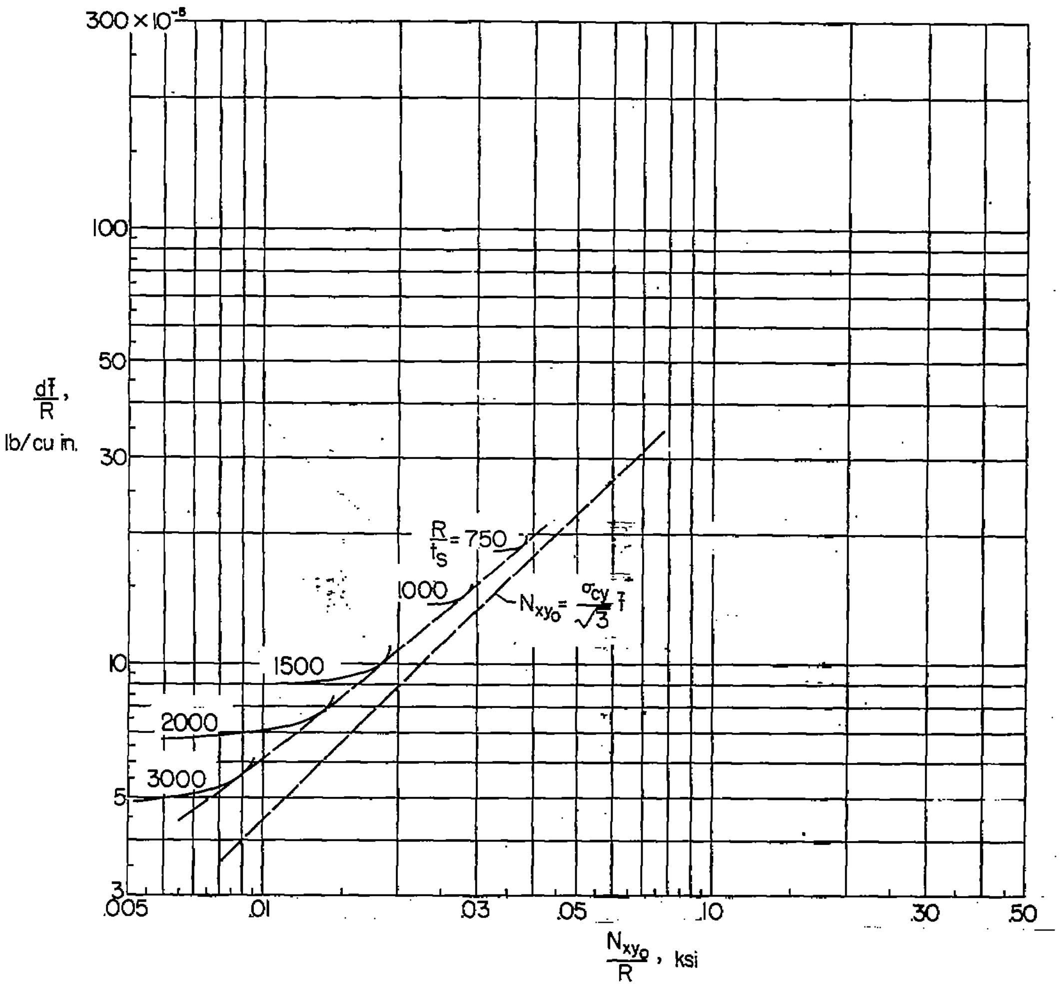

Figure 20.- Weight-strength plot of AZ31A-H24 magnesium-alloy sandwich-type cylindrical shells subjected to shear stresses. $\delta = 0.03$; $l/R = 1.0$.

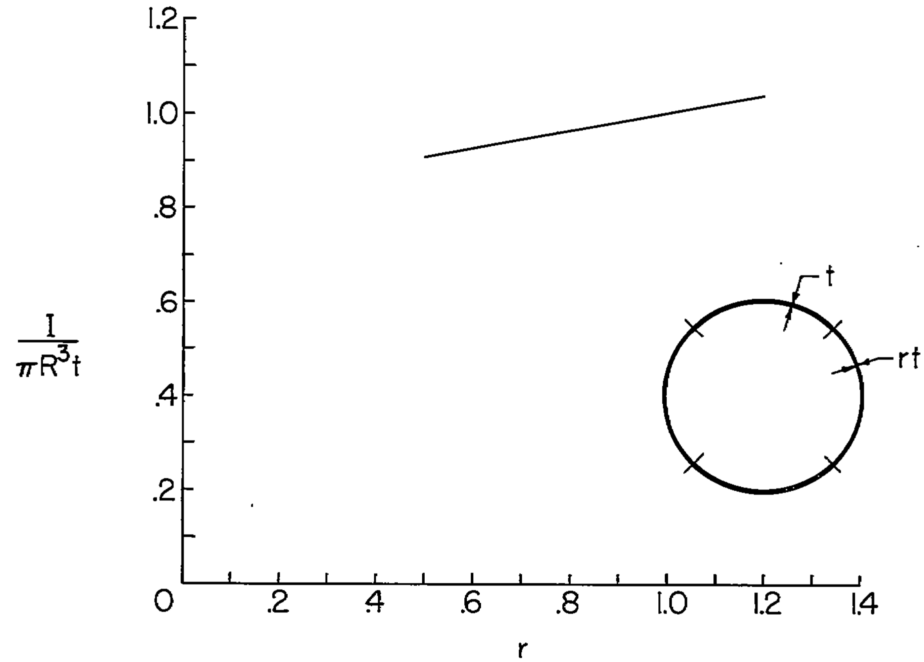

Figure 21.- Effect of changes in shear area on stiffness of circular cylinders.

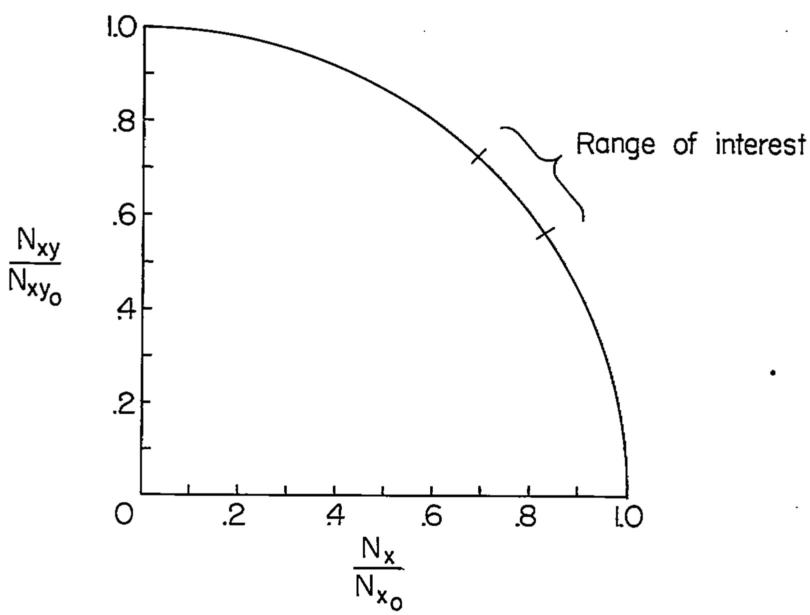

Figure 22.- Interaction diagram for cylinders subjected to shear and bending, showing loading range of interest in the present study.

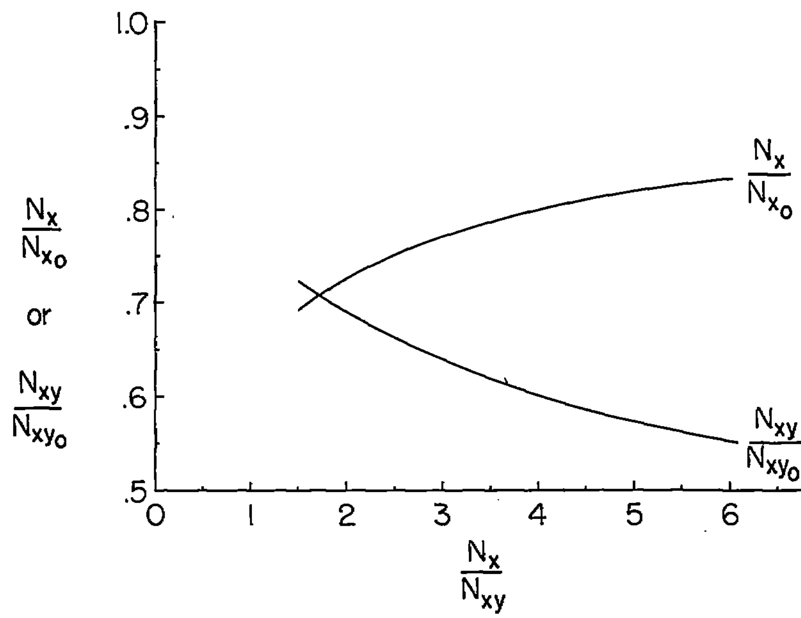

Figure 23.- Effect of combined bending and shear loads on the strength of cylindrical shells.

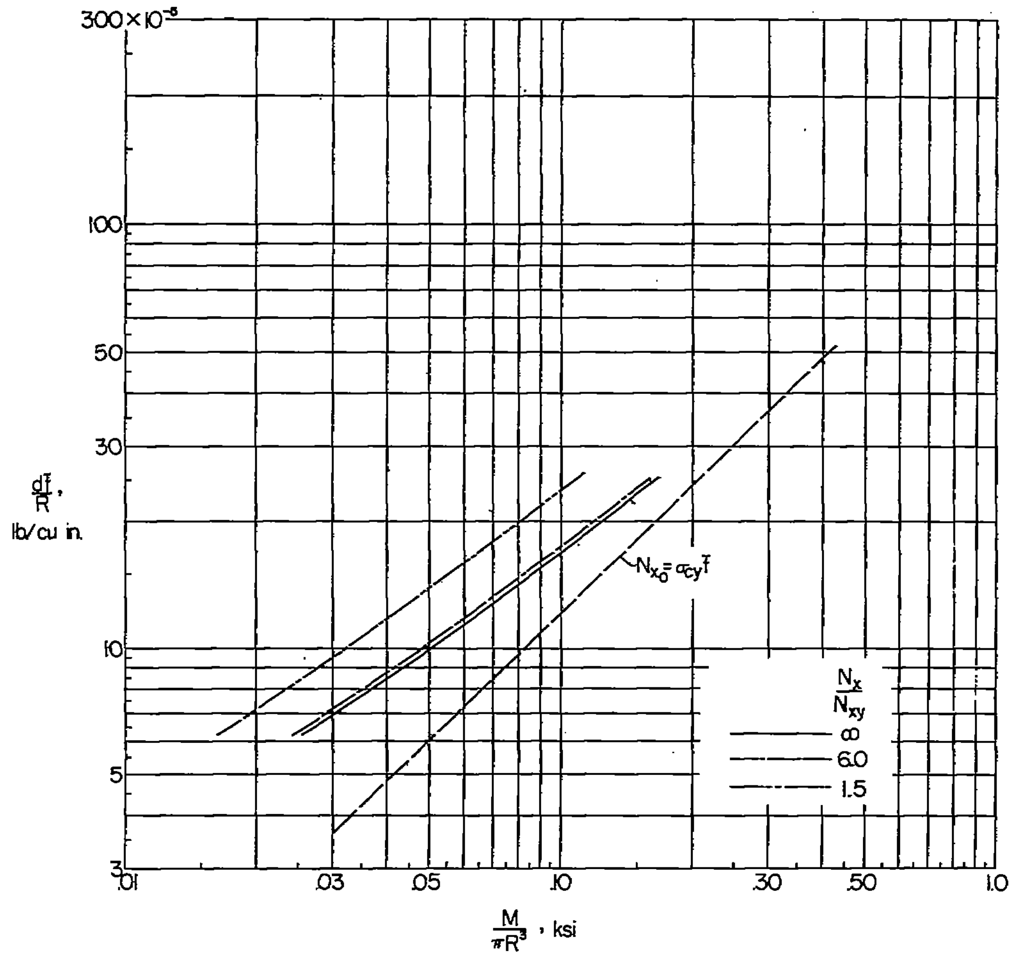

Figure 24.- Weight-strength comparisons for 7075-T6 aluminum-alloy sandwich-type cylindrical shells subjected to two ratios of bending stress to shear stress. Wall thickness of cylinder is constant around circumference. $\delta = 0.03$; $l/R = 1.0$.

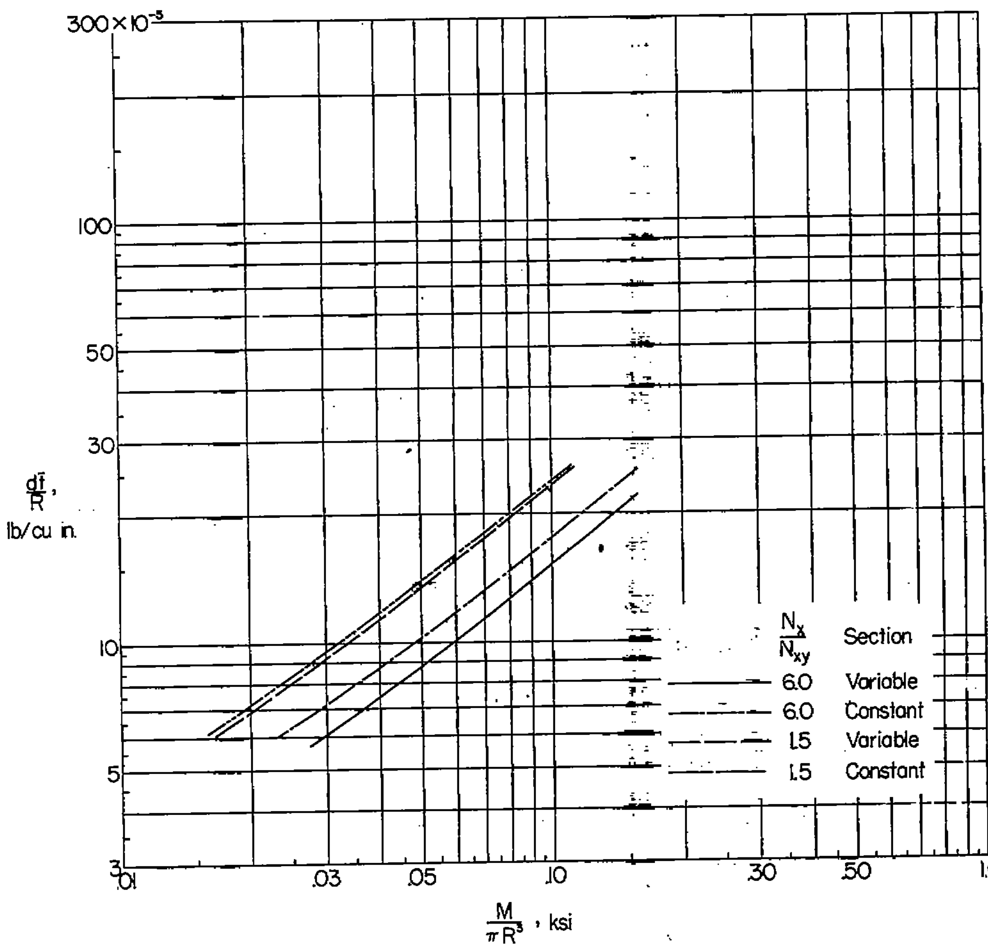

Figure 25.- Weight-strength comparisons for 7075-T6 aluminum-alloy sandwich-type cylindrical shells subjected to two ratios of bending stress to shear stress. Wall thickness of cylinder varies around circumference. $\delta = 0.03$; $l/R = 1.0$.

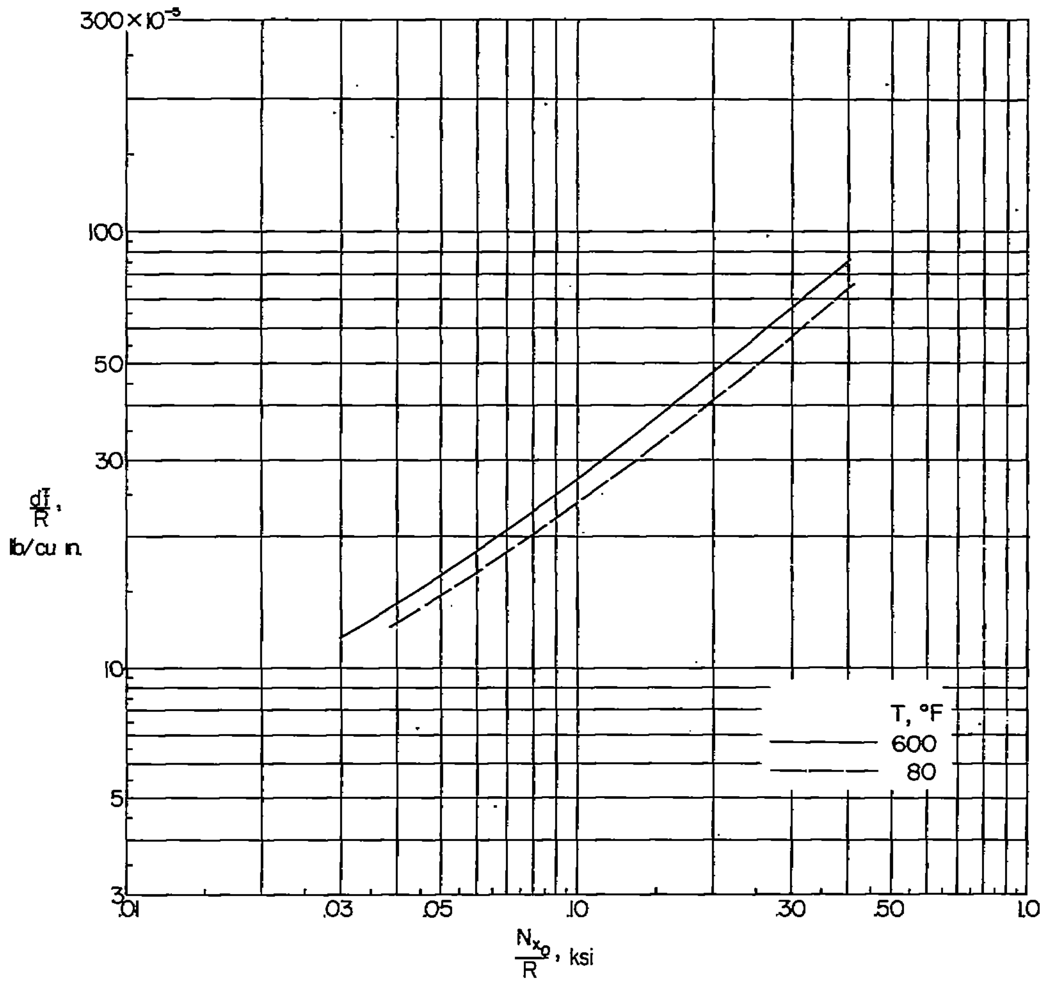

Figure 26.- Weight-strength comparison for 17-7 PH stainless-steel sandwich-type cylindrical shells subjected to bending stresses for two values of temperature.